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TRIGONOMETRY:

ANALYTICAL, PLANE AND SPHERICAL.

WITH

LOGARITHMIC TABLES.

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PREFACE.

AN author of a new trigonometry, at the present time, owes an explanation if not apology, both to teachers and students, for adding another to the already too numerous works upon this subject. The author was frequently applied to for opinions upon works extant and those proposed for publication, but found none that quite suited him; so the present work was undertaken to provide for his own classes and avoid further annoyance in being called upon to criticise others, even though it furnishes one more book for other critics to pass upon. The author found, however, when he had reduced his ideas to a form ready for print, that he did not differ from certain other writers as much as he, at first, expected to; still the many distinctive characteristics which remain will, we trust, commend themselves to others.

Attention is called to the fact that the trigonometrical functions are first defined and treated as ratios, but that afterwards they are represented by lines, which lines are so defined as to represent the functions. The latter may be made more prominent by any instructor

who desires so to do, although the chief object in the plan of the work was to furnish an aid to the memory for those who can remember geometrical representations more easily than abstract ratios.

We trust that the new notation in article 32 will meet with favor, as its adoption will remove an ambiguity which has thus far existed in this science. The functions as defined in article 16 are not restricted to quadrants, and their treatment afterwards in articles 23 to 30 are perfectly general, and hence it seems unnecessary after deducing equation (30) to make other and special demonstrations for particular limitations of the angles x and y. Similar remarks apply to equations following equation (30). Imaginary, or impossible functions have been introduced for the purpose of affording a greater variety in the exercises.

AUTHOR.

HOBOKEN, 1885.

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TRIGONOMETRY.

CHAPTER I.

FUNDAMENTAL PRINCIPLES.

1. *A geometrical quantity* is any form, figure, or magnitude conceived of in space.

A geometrical point is a figure from which magnitude is abstracted. It has no size and simply marks a place.

A geometrical line is that property of a figure which has length only; or from which breadth and thickness are abstracted.

A line may be generated by the movement of a point. If the generating point preserves a constant direction, the line generated is a *right line*, but if it changes its direction at every point, it generates a *curved line*. A right line is always understood unless otherwise stated. A right line is said to be given when its length and direction are known in reference to some other line assumed to be fixed.

2. **Angles.** *An angle* is the difference in direction of two lines. It is measured by means of some angle arbitrarily assumed for the unit. In reference to a figure representing an angle, it may be considered as the opening between two lines; the opening being greater as the inclination between the lines is greater.

Thus the direction or inclination of the line OF compared with that of OE is the angle between them. An angle is read by means of letters as EOF, the letter O at the vertex being the middle letter; but sometimes it is referred to by simply naming the letter at the vertex as O.

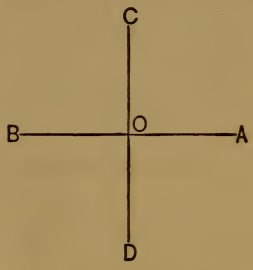
An angle may be denoted by a single letter, as x, y, etc., or by a Greek letter as α, β, γ . . . θ, φ, etc., in which case we have

$$x = EOF, \quad . \quad . \quad . \quad \text{or } \varphi = EOF, \quad . \quad . \quad . \quad \text{etc.}$$

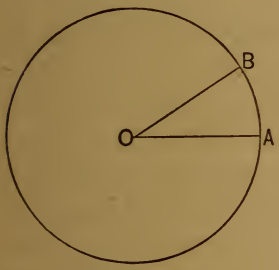
An angle may be generated by revolving a line about a point. Thus, the angle EOF may be conceived to be generated by revolving a line from OE into the final position of OF. The line OE from which OF starts in the rotation, is called the *initial line*, and OF in its final position, the *terminal line*. The moving line, OF, is called the *generatrix*. The point O is the *origin*, *vertex*, or *pole*. An angle in this case measures the amount of rotation of the generating line.

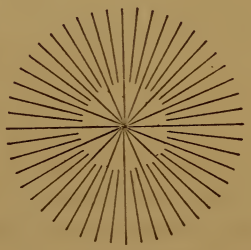
If two lines do not intersect, they are conceived to make the same angle as two other lines passing through a point and parallel respectively to the given lines. Thus, the angle between the lines AB and CD, whether in the same plane or not, is the same as between the lines OE and OF, drawn through the point O and parallel respectively to CD and AB.

3. **Measurement of Angles.** If two lines, AB and CD, so intersect as to make the four angles at O equal, the lines will be mutually perpendicular, and each of the angles is called a *right angle.* A right angle is sometimes taken as a unit angle. An acute angle is less than a right angle, and an obtuse angle greater.



An angle may be measured by the arc subtending it, the centre of the arc being at the vertex of the angle. For, the radius being constant, the angle and arc will be generated by the same amount of rotary motion of the generating line OB, and of the point B. In order to compare angles by means of arcs, the radius must be constant, and, unless otherwise stated, it will be considered as unity. Then will π be the length of a semi-circumference; and $\frac{1}{2}\pi$ of one-fourth of a circumference and hence will subtend a right angle, π will subtend two right angles, $\frac{3}{2}\pi$ three right angles and, generally, $\frac{1}{2}n\pi$, n right angles, n being any integer. We say, for brevity, $\frac{1}{2}\pi$ is a right angle, $\frac{1}{4}\pi$ is one-half of a right angle, etc. This method of measuring an angle is called *circular measure.*

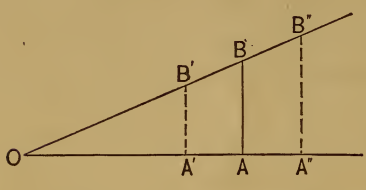




If 360 lines radiate from a point, making equal angles between the consecutive lines, each

angle is, by common consent, called *a degree*. This is the unit-angle commonly employed in practice.

Hence, we say, that the complete revolution of a line about a point generates 360°, and that in one right angle there are 90 degrees. Also an arc $\frac{1}{2}\pi$ subtends an angle of 90 degrees, $\frac{1}{4}\pi$ of 45 degrees, and so on. And for brevity we sometimes say, $\frac{1}{2}\pi$ equals 90 degrees, $\frac{1}{4}\pi$ equals 45 degrees, etc.



An angle may also be measured by means of certain other lines bounding it. Thus, if from any points B', B, B'', etc., in the line OB, perpendiculars, $B'A'$, BA, $B''A''$, etc., be let fall upon OA we have from similar triangles

$$\frac{A'B'}{OA'} = \frac{AB}{OA} = \frac{A''B''}{OA''} = a\ constant$$

for any fixed angle; hence, if this ratio be determined in any manner, it may be used as a measure of that angle. Thus, this ratio for an angle $AOB = 45$ degrees, will be unity; hence, conversely, when this ratio is 1 we know that the angle O is 45 degrees. The ratio for other angles may be found and the results tabulated. We also have

$$\frac{OA'}{OB'} = \frac{OA}{OB} = a\ constant$$

for a given angle. Similarly, ratios may be found between other lines about the angle. This mode of measuring angles forms the basis of the science which we are to consider.

$(\sin A)^n$, or $\sin^n A$; the latter being the more common notation; and similarly for other functions. The sine of the nth power of the angle is written $\sin A^n$; and similarly for the other functions. The sine of n times an angle is written $\sin (nA)$, or simply $\sin nA$; and similarly for the other functions. The reciprocal of the sine of A is written $(\sin A)^{-1}$ or $\frac{1}{\sin A}$, but not $\sin^{-1} A$, the latter notation being often employed for another purpose. The functions may be multiplied and divided in the usual manner; thus $\sin A \cos A$ implies that $\cos A$ is multiplied by $\sin A$, and similarly for the others.

6. One quantity is said to be a function of another when it is so related to the other that a change in the value of the latter causes a change in the value of the former, thus in the expressions

$$y = 3x^2 - ax, \qquad y = \log x, \qquad y = \sin x, \qquad y = \tan x,$$

y is said to be a function of x; and the first is called an algebraic function, the second is logarithmic, and the others trigonometrical. All functions not *algebraic* are called *transcendental.* When the form of a function is unknown, or when brevity alone is desired, it may be written

$$y = F(x), \qquad \text{or } y = f(x), \qquad \text{or } y = \varphi(x);$$

all of which are read "y is a function of x." In these expressions F, f, φ, are not quantities but symbols implying some *form* of an expression.

7. *Trigonometry* is that science which treats of the

properties and relations of trigonometrical functions, and of their use in the solution of triangles. It is considered in three parts, viz. :

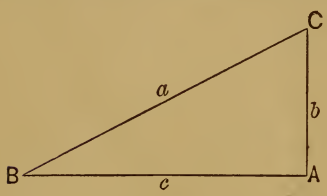
Analytical Trigonometry treats of the abstract properties of trigonometrical functions.

Plane Trigonometry treats of the solution of plane triangles by means of trigonometrical functions.

Spherical Trigonometry treats of the solution of spherical triangles by means of trigonometrical functions.

EXERCISES.

1. In the right triangle BAC, if $BA = 4$, $AC = 3$, what will be the sine of the acute angles ?



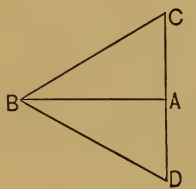
[We have $BC = \sqrt{4^2 + 3^2} = 5$; hence, according to the definition, $\sin B = \frac{3}{5}$ and $\sin C = \frac{4}{5}$.]

2. Find the value of the other seven trigonometrical functions of B in the preceding exercise.

3. Find the trigonometrical functions of 45°.

In this case $BA = AC$, and $BC = BA\sqrt{2}$. It will be found that $\sin 45° = \frac{1}{2}\sqrt{2} = \cos 45°$; $\tan 45° = 1 = \cot 45°$; $\sec 45° = \sqrt{2} = \csc 45°$.

4. Find the trigonometrical functions of 30°.



Let CBD be an equilateral triangle, then will each of the angles be 60°. From B drop the perpendicular BA, then will $BAC = 90°$, $CBA = 30°$, $AC = AD = \frac{1}{2}CD = \frac{1}{2}BC$;

$$\therefore \sin 30° = \frac{AC}{BC} = \frac{\frac{1}{2}BC}{BC} = \frac{1}{2} ; \cos 30° = \frac{BA}{BC} = \frac{\sqrt{BC^2 - AC^2}}{BC} = \frac{\sqrt{4AC^2 - AC^2}}{2AC} = \frac{1}{2}\sqrt{3} ;$$

$$\tan 30° = \sqrt{\tfrac{1}{3}} ; \quad \cot 30° = \sqrt{3}.$$

5. Find the trigonometrical functions of 60°.
If $B = 30°$ in the right triangle BAC, then will $C = 60°$.

$$\sin 60° = \frac{AB}{CB} = \tfrac{1}{2}\sqrt{3} = \cos 30° ; \qquad \cos 60° = \tfrac{1}{2} = \sin 30° ;$$

$$\tan 60° = \sqrt{3} = \cot 30° ; \qquad \cot 60° = \tfrac{1}{3}\sqrt{3} = \tan 30°.$$

6. Show that $\sin^2 A + \cos^2 A = 1$.
[We have from group (*A*)

$$\sin^2 A + \cos^2 A = \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{p^2 + b^2}{h^2} = \frac{h^2}{h^2} = 1.]$$

7. Show that $\sec^2 A = 1 + \tan^2 A$.

8. Show that

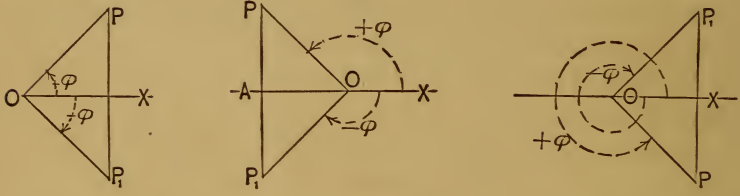
$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}} = \frac{1}{\sqrt{\csc^2 A - 1}}.$$

Prove the following equations :

9. $\tan A \cot A = 1$.
10. $\cos A = \sin A \cot A$.
11. $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$.
12. $\sin^2 A - \cos^2 B = \sin^2 B - \cos^2 A$.
13. $\tan A + \cot A = \sec A \csc A$.
14. $(\sin^2 \theta + \cos^2 \theta)^n = 1$.
15. $(\sin^2 \varphi - \cos^2 \varphi)^2 = 1 - 4\cos^2 \varphi + 4\cos^4 \varphi$. (Observe that from Example 6 we have $\sin^2 \varphi = 1 - \cos^2 \varphi$.)
16. $\dfrac{\tan \varphi + \tan \theta}{\cot \varphi + \cot \theta} = \tan \varphi . \tan \theta$.
17. $3 \sin 60° - 4 \sin^3 60° = 4 \cos^3 30° - 3 \cos 30°$.
18. $\sin^2 (\theta + \varphi) + \cos^2 (\theta + \varphi) = 1$.
19. $\sin 45° + \cos 30° = \tfrac{1}{2}(\sqrt{2} + \sqrt{3})$.
20. $\tan 30° \cdot \cos 45° = \tfrac{1}{2}$.
21. $\dfrac{1 - \sin \varphi}{1 + \sin \varphi} = (\sec \varphi - \tan \varphi)^2$.
22. $\sin 60° \cdot \cot 60° = \tfrac{1}{2}$.
23. $(\tan A + \cot A) \sin A \cdot \cos A = 1$.
24. $\tan 30° \cdot \tan 60° \cdot \cos 45° \cdot \sec 45° = 1$.
25. $\cos x \cdot \tan x = \sin x$.
26. $\csc x \cdot \tan x = \sec x$.

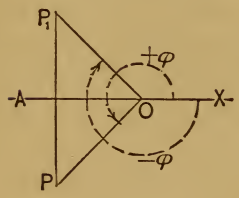
Analytical Trigonometry.

8. We now consider the principles according to which the functions may be so extended as to be more general. The algebraic signs + and − applied to a line indicate opposite directions. Thus, if a line generated by a point moving in one direction be *positive*, the opposite direction will be *negative;* the former of which will be indicated by + and the latter by −. If AB be positive BA and AC will be negative.



9. *Angles and arcs* may also be affected by the signs + and −. Thus, if the angle XOP, generated by a

left-handed rotation, be considered *positive*, the angle XOP_1, generated by a right-handed rotation, will be *negative*. The signs + and − indicate relative direction of motion in generating a magnitude.



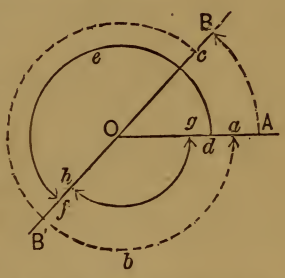
The angle POX would also be negative, the order of the letters indicating that the rotation is from P to X, or right handed. The positive arc is here indicated by $+\varphi$, and the negative arc by $-\varphi$.

Since the above relations are arbitrary, either may be assumed as positive, but when once chosen it must be

used in that sense through that discussion. According to custom, left-handed rotation will be understood unless otherwise stated.

10. *Theoretically, angles have no limits* as to size, for the generating line may be revolved about the pole any number of times; still all possible directions in a plane will have been passed over by one revolution of the radius.

If φ be an angle less than 360°, the terminal position of the generating line will be the same for the angles φ, $2\pi + \varphi$, $4\pi + \varphi$ . . . $2n\pi + \varphi$, where n is zero or an integer either positive or negative.



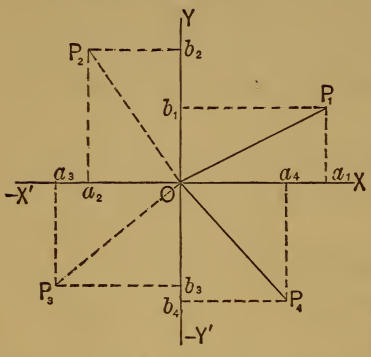
11. If OB be the terminal line, any point in it, as B, may be determined by any one of the four following ways: By revolving a line from OA in a positive direction through an angle AOB until it passes through the point B, when it will be determined by the angle AOB and distance OB. Or, by revolving the generating line *negatively* through the angle *abc* until it passes through B. Or, by revolving it *positively* through the angle *def* into the position OB', so that if prolonged negatively through O it will pass through B. Or, by revolving it negatively through the angle *gh* into the latter position.

To represent these algebraically, let ρ = the distance OB, and $\varphi = AOB$; then the point B may be found by either one of the four following combinations:—

$$\left.\begin{matrix} +\rho \\ +\varphi \end{matrix}\right\}; \quad \left.\begin{matrix} +\rho \\ -(360 - \varphi) \end{matrix}\right\}; \quad \left.\begin{matrix} -\rho \\ +(180 + \varphi) \end{matrix}\right\}; \quad \left.\begin{matrix} -\rho \\ -(180 - \varphi) \end{matrix}\right\}.$$

The negative radius, $-\rho$, is unnecessary in this science, and hence will not be further considered, but both positive and negative angles will be used.

12. **Coordinates.** Two mutually perpendicular lines XX', YY', when used as lines of reference, are called coördinates, and the lines are called *coördinate axes.* The horizontal line XX' is called the axis of *abscissas,* and the perpendicular line YY' is called the axis of *ordinates.* The angle XOY is the first quadrant; YOX', the second; $X'OY'$, the third; and $Y'OX$, the fourth quadrant.



Coördinate axes may be oblique, but as used in this work it should be observed that they must be mutually perpendicular.

13. *The abscissa* of a point is its distance from the axis of ordinates YY', measured on a line parallel to the axis of abscissas. The abscissas to the right of YY' will be considered *positive,* those to the left *negative.* Thus, the abscissa of P_1 will be $+ b_1P_1$; of P_2, $- b_2P_2$; of P_3, $- b_3P_3$; and of P_4, $+ b_4P_4$. The letters b_1, b_2, etc., are placed before P_1, P_2, etc., so that the order of the letters will indicate the direction of P_1, P_2, etc., from the axis YY'.

14. *The ordinate* to any point is its distance from the axis of abscissas measured on a line parallel to the axis of ordinates. The ordinates above XX' will be considered *positive,* those below, *negative.* Thus, the ordi-

nate of P_1 will be $+ a_1P_1$; of P_2, $+ a_2P_2$; of P_3, $- a_3P_3$; of P_4, $- a_4P_4$.

The abscissa and ordinate of a point are together called its *coördinates.*

15. *The signs of the coördinates* in the four quadrants will be

	1st Quadrant.	2d Quadrant.	3d Quadrant.	4th Quadrant.
Abscissa,	+	−	−	+
Ordinate,	+	+	−	−

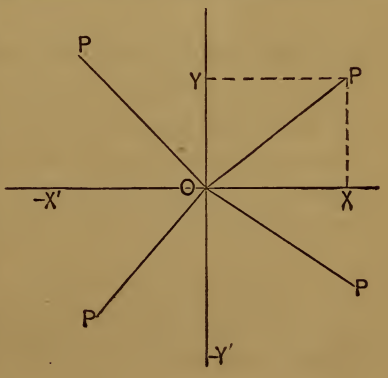
EXERCISES.

Show in what quadrant the revolving line will be when it has described the angles, n being an integer :

1. $120°$	2. $390°$	3. $490°$
4. $700°$	5. $-120°$	6. $-490°$
7. $\frac{2}{3}\pi$	8. $10\frac{1}{4}\pi$	9. $-8\frac{1}{4}\pi$.
10. $2n\pi$	11. $(2n+\frac{1}{2})\pi$	12. $-(4n-\frac{1}{4})\pi$.
13. $(6n+1)\frac{\pi}{3}$.	14. $(6n-2)\frac{\pi}{3}$	15. $-(8n+2)\frac{\pi}{4}$..

---

16. Letting XOP be any angle x, O the origin of coördinates, $OP = r$, the radius vector, or simply the radius, being the distance from the origin to any point P in the terminal side of the angle x, $XP = o$, the ordinate to the point P, $YP = a$, the abscissa to the same point, where r, o, a, are initial letters of the quantities



they respectively represent; we define the trigonometrical functions in a more general way, as follows:

$$\left.\begin{aligned}
\sin x &= \frac{ordinate}{radius} &&= \frac{XP}{OP} = \frac{o}{r} \\
\cos x &= \frac{abscissa}{radius} &&= \frac{YP}{OP} = \frac{a}{r} \\
\tan x &= \frac{ordinate}{abscissa} &&= \frac{XP}{OX} = \frac{o}{a} \\
\cot x &= \frac{abscissa}{ordinate} &&= \frac{OX}{XP} = \frac{a}{o} \\
\sec x &= \frac{radius}{abscissa} &&= \frac{OP}{OX} = \frac{r}{a} \\
\csc x &= \frac{radius}{ordinate} &&= \frac{OP}{XP} = \frac{r}{o} \\
\text{vsn } x &= \frac{r - abscissa}{radius} &&= \frac{r-a}{r} \\
\text{cvs } x &= \frac{r - ordinate}{radius} &&= \frac{r-o}{r}
\end{aligned}\right\} (B).$$

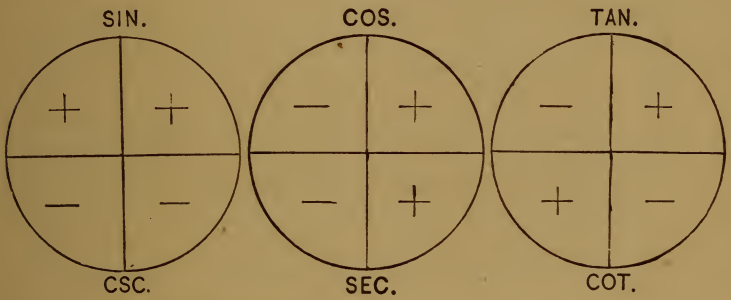
in which o and a may be plus or minus.

The definitions on page 6, were, strictly speaking, for an acute angle, but by giving + and − signs to o and a as may be necessary, the definitions in group (B) are general, and applicable to any angle from OX to the terminal position of the generating line.

17. *Signs of the functions.* The signs of the co-ordinates given in article 15, applied to group (B), gives for the signs of the functions for the several quadrants:

Functions.	1st quad.	2d quad.	3d quad.	4th quad.
sin and csc	+	+	−	−
cos and sec	+	−	−	+
tan and cot	+	−	+	−
vsn and cvs	+	+	+	+

The sine and cosine have like signs in the 1st and 3d quadrants, and the tangent and cotangent are positive in the same quadrants.



18. *Limiting values of the ordinate and abscissa.* For zero degrees, the terminal and initial lines coincide, hence the ordinate will be zero, and the abscissa equal r. Symbolically, we have for $x = 0°$, $a = r$, and $o = 0$. In this way we find from the figure that for

$$\begin{aligned}
x &= \ \ 0°, & a &= \ \ r, & o &= \ \ 0,\\
x &= \ 90°, & a &= \ \ 0, & o &= \ \ r,\\
x &= 180°, & a &= -\,r, & o &= \ \ 0,\\
x &= 270°, & a &= \ \ 0, & o &= -\,r,\\
x &= 360°, & a &= \ \ r, & o &= \ \ 0;
\end{aligned}$$

the last being the same as the first, and after this the values repeat themselves at the end of every quadrant.

19. **Limiting values of the functions.** The values of a and o given in the preceding article, substituted in group (B) give

Functions.	Angles.			
	0°	90°	180°	270°
sin	∓ 0	1	± 0	-1
cos	1	± 0	-1	∓ 0
tan	∓ 0	$\pm \infty$	∓ 0	$\pm \infty$
cot	$\mp \infty$	± 0	$\mp \infty$	± 0
sec	1	$\pm \infty$	-1	$\mp \infty$
csc	$\mp \infty$	1	$\pm \infty$	-1
vsn	$+0$	1	2	1
cvs	1	0	1	2

[The order of the signs $\pm$ indicates that the function has passed from $-$ to $+$; that is just before the generating line reached the given angle the sign of the function was $-$, and was $+$ immediately after passing it.]

20. **Relative values of the functions.** From group (B) we derive the following:

$$\frac{\sin x}{\cos x} = \frac{\frac{o}{r}}{\frac{a}{r}} = \frac{o}{a} = \tan x, \tag{1}$$

$$\frac{\cos x}{\sin x} = \frac{a}{o} = \cot x; \tag{2}$$

that is, *The tangent of any angle equals the ratio of the sine of that angle to its cosine;* and

The cotangent of an angle equals the ratio of the cosine to the sine.

Also from the same group we find the following combinations of the functions equal to unity:

$$\sin^2 x + \cos^2 x = \frac{a^2 + o^2}{r^2} = \frac{r^2}{r^2} = 1, \tag{3}$$

$$\tan x \cot x = 1, \tag{4}$$

$$\sin x \csc x = 1, \tag{5}$$

$$\cos x \sec x = 1, \tag{6}$$

$$\cos x + \text{vsn } x = 1, \tag{7}$$

$$\sin x + \text{cvs } x = 1. \tag{8}$$

21. *Each function may be expressed in terms of any one of the other seven.* Thus, from equations (3), (5), (8), of the preceding article we have directly

$$\sin x = \sqrt{1 - \cos^2 x} = \frac{1}{\csc x} = 1 - \text{cvs } x,$$

which gives the value of $\sin x$ in terms of cos, csc, and cvs. To find it in terms of the tangent, we have from (1),

$$\sin x = \tan x \cos x$$

from (3),

$$= \tan x \sqrt{1 - \sin^2 x};$$

$$\therefore \sin^2 x = \tan^2 x - \tan^2 x \sin^2 x,$$

transposing,

$$(1 + \tan^2 x) \sin^2 x = \tan^2 x;$$

$$\therefore \sin x = \frac{\tan x}{\sqrt{1 + \tan^2 x}}; \tag{9}$$

which is the required result.

From (4) we have

$$\tan x = \frac{1}{\cot x}, \tag{10}$$

which substituted in the preceding equation gives

$$\sin x = \frac{1}{\sqrt{1 + \cot^2 x}};$$

and so on for the other two functions. In this way the following table may be formed.

Given Functions.	Required Functions.					
	$\sin$	$\cos$	$\tan$	$\cot$	$\sec$	$\csc$
$\sin$	$\sin$	$\sqrt{1 - \cos^2}$	$\frac{\tan}{\sqrt{1 + \tan^2}}$	$\frac{1}{\sqrt{1 + \cot^2}}$	$\frac{\sqrt{\sec^2 - 1}}{\sec}$	$\frac{1}{\csc}$
$\cos$	$\sqrt{1 - \sin^2}$	$\cos$	$\frac{1}{\sqrt{1 + \tan^2}}$	$\frac{\cot}{\sqrt{1 + \cot^2}}$	$\frac{1}{\sec}$	$\frac{\sqrt{\csc^2 - 1}}{\csc}$
$\tan$	$\frac{\sin}{\sqrt{1 - \sin^2}}$	$\frac{\sqrt{1 - \cos^2}}{\cos}$	$\tan$	$\frac{1}{\cot}$	$\sqrt{\sec^2 - 1}$	$\frac{1}{\sqrt{\csc^2 - 1}}$
$\cot$	$\frac{\sqrt{1 - \sin^2}}{\sin}$	$\frac{\cos}{\sqrt{1 - \cos^2}}$	$\frac{1}{\tan}$	$\cot$	$\frac{1}{\sqrt{\sec^2 - 1}}$	$\sqrt{\csc^2 - 1}$
$\sec$	$\frac{1}{\sqrt{1 - \sin^2}}$	$\frac{1}{\cos}$	$\sqrt{1 + \tan^2}$	$\frac{\sqrt{1 + \cot^2}}{\cot}$	$\sec$	$\frac{\csc}{\sqrt{\csc^2 - 1}}$
$\csc$	$\frac{1}{\sin}$	$\frac{1}{\sqrt{1 - \cos^2}}$	$\frac{\sqrt{1 + \tan^2}}{\tan}$	$\sqrt{1 + \cot^2}$	$\frac{\sec}{\sqrt{\sec^2 - 1}}$	$\csc$

EXERCISES.

1. In what quadrants may A be if $\sin A = -0.567$?
2. If $\sin x = -\frac{1}{2}$, find the other functions when the terminal line is in the third quadrant.
3. Find the functions when $\tan x = 2$, and the terminal line is in the third quadrant.
4. Find x when $\sin x = \cos x$.

   [Make $\cos x = \sqrt{1 - \sin^2 x}$, solve and compare the result with exercise 3, page 8.]

5. Find x when $\sin x = \tan x$. *Ans.* $x = 0$.
6. Find the trigonometrical functions corresponding to $\cot x = 2$.
7. If $\tan x = 2 \cos x$, find $\sin x$ in terms of $\cot x$.

   [$\tan x = \dfrac{1}{\cot x}$, and $\cos x = \sqrt{1 - \sin^2 x}$; hence we find

   $$\sin x = \sqrt{1 - \frac{1}{4 \cot^2 x}}\ ]$$

8. If $\cot x = 2 \tan x$, find $\sin x$ in terms of $\cos x$.
9. Find $\sin x$ from the equation $2 \sin^2 x - 3 \sin x = 1$.

   [Here $\sin x$ is the unknown quantity, and the equation is to be solved as a complete quadratic.]

10. Find $\tan x$ from the equation $a \tan x + b \cot x = c$.

    [Substitute $\cot x$ from equation (4) and solve.]

11. Find $\sin x$ from the equation $3 \sin x - 2 \cos x = 2 \operatorname{vsn} x$.

    Ans. $\frac{2}{3}$.

12. Show that $\sin^2 \frac{1}{2}x + \cos^2 \frac{1}{2}x = 1$.
13. Show that $\sin^2 nx = 1 - \cos^2 nx$.
14. What does $\sin^2 (1 - \frac{1}{2}x) + \cos^2 (1 - \frac{1}{2}x)$ equal ?
15. In a right triangle, if the base be $3a$ and the perpendicular $4a$, find each of the eight trigonometrical functions.
16. Given $2 \sin x = 3 \cos y$, and $\tan y = 3 \cos x$, to find $\sin x$.

    [We have from equation (1) $\tan y = \dfrac{\sin y}{\cos y}$, which, by means of equation (3), becomes $\dfrac{\sqrt{1 - \cos^2 y}}{\cos y}$, and this in the second of the given equations gives

    $$1 - \cos^2 y = 9 \cos^2 x \cos^2 y,$$

from which finding $\cos y$ and substituting in the first of the given equations, gives $\sin x = \pm 0.89 +$ or $\pm 0.56 +$.]

17. Given $\sin x + \cos y = 1$, and $\sin x \cos y = -\frac{1}{2}$, find $\sin x$ and $\sin y$.
18. Solve the simultaneous equations $\tan x \cot y = 2$, and $\sin x \cos y = \frac{1}{4}$.
19. Find $\sin x$ from the equations $a^{\sin x} = b^{\cos y}$, and $\sin^a x = \cos^b y$.

[From the first equation, $\sin x \log a = \cos y \log b$; from the second, $\cos y = (\sin x)^{\frac{a}{b}}$ Substituting gives

$$\sin x = \frac{\log b}{\log a}(\sin x)^{\frac{a}{b}}, \quad \text{or } \sin x \left[1 - \frac{\log b}{\log a}(\sin x)^{\frac{a}{b}-1}\right] = 0;$$

$$\therefore \sin x = 0, \text{ or } \sin x = \left(\frac{\log a}{\log b}\right)^{\frac{b}{a-b}}.]$$

20. Find y from the equations $(\tan x)^{\cot y} = 5$, $(\tan x)^{\cos y} = \frac{1}{2}$.
21. Find the values of x from the equation

$$\sin x \cos \tfrac{1}{2}x \tan \tfrac{1}{4}x = 0.$$

[Each factor may be zero, hence $\sin x = 0$, and $x = 0, \pi, 2\pi$, &c.
If $\cos \frac{1}{2}x = 0$, then $\frac{1}{2}x = \frac{1}{2}\pi, \frac{3}{2}\pi$, &c., and $x = \pi, 3\pi, 5\pi$, &c.
If $\tan \frac{1}{4}x = 0$, then $\frac{1}{4}x = 0, \pi, 2\pi$, &c., and $x = 0, 4\pi, 8\pi$, &c.]

---

22. **Functions of negative angles.** If the angle be negative, and numerically less than 90°, it will be below the line OX in the figure on page 21, and the ordinate o will be negative, or $-o$, while the abscissa will be positive, or $+a$; and we have

$$\sin(-x) = \frac{-o}{r} = -\frac{o}{r} = -\sin x. \tag{11}$$

Similarly,

$$\cos(-x) = \frac{a}{r} = \cos x. \tag{12}$$

It may be shown that the same relation exists when the angle exceeds 90°; hence we have generally,

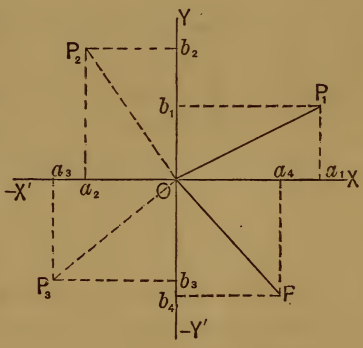
$$\cos(-x) = \cos x, \qquad \sec(-x) = \sec x, \tag{13}$$

$$\left.\begin{aligned} \sin(-x) &= -\sin x, & \tan(-x) &= -\tan x, \\ \csc(-x) &= -\csc x, & \cot(-x) &= -\cot x. \end{aligned}\right\} \tag{14}$$

That is, *The cosine and secant of a negative angle are identical with that of a positive angle of the same magnitude; but the sine, cosecant, tangent, and cotangent of a negative angle are* MINUS *the like-named function of an equal positive angle.*

23. *The complement* of an angle is "90° *minus* the angle." Thus the complement of the angle XOP_1 will be $90 - XOP_1 = P_1OY$; of XOP_2, $90° - XOP_2 = -YOP_2$; and so on.

The complement of an angle exceeding 90° will be negative, and the complement of a negative angle will be positive, and exceed 90°.

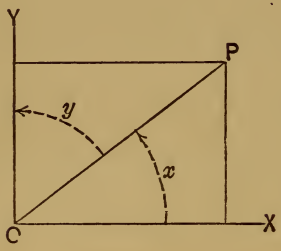


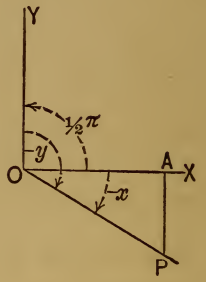
24. *The supplement* of an angle is "180° *minus* the angle." Thus the supplement of XOP_2 will be $180° - XOP_2 = P_2OX'$.

25. *Complementary functions.* Let $XOP = x$, and $POY = y$; then, because of alternate angles (Fig. on p. 22),

$$x = XOP = OPY = 90° - y.$$

The angle x may be produced in this case, by revolving the generating line positively 90°, or to OY, then negatively through an angle $y = YOP$, giving $x = XOP$.



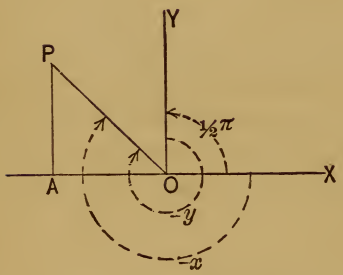


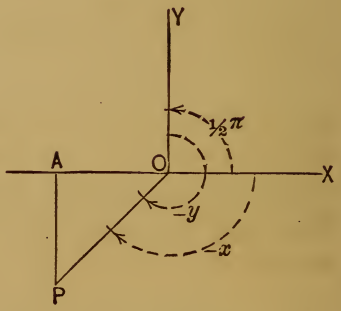
From the figure and group (B) we have

$$\frac{o}{r} = \sin XOP = \sin x = \sin (90° - y),$$

and $$\frac{o}{r} = \cos POY = \cos y;$$

$$\therefore \sin XOP = \cos POY;$$





and this is true for any value of y; and corresponding results may be found for the other functions; hence we have for *the functions of* $90° - y$,

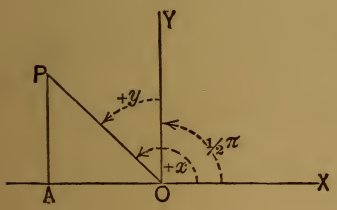
$$\left.\begin{array}{ll} \sin(90^\circ - y) = \cos y. & \cos(90^\circ - y) = \sin y. \\ \tan(90^\circ - y) = \cot y. & \cot(90^\circ - y) = \tan y. \\ \sec(90^\circ - y) = \csc y. & \csc(90^\circ - y) = \sec y. \end{array}\right\} (16)$$

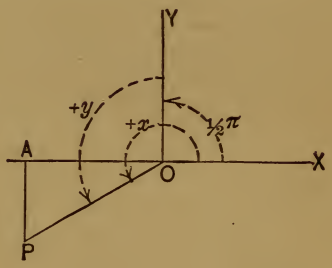
That is, *The sine, tangent, or secant of an angle equals the co-function of its complement.*

[Cosine may be considered as an abbreviation for "the sine of the complement of an angle," and similarly for cotangent and cosecant.]

26. *Functions of* $90^\circ + y$.

Writing $-y$ for y in the first of equations (16), the left member will become $\sin x = \sin[90^\circ - (-y)] =$





$\sin[90^\circ + y]$, and the right member, $\cos(-y)$, which being reduced by the first of equations (14) becomes $\cos y$; and this is true whatever be the magnitude of y. Treating the other functions in the same manner, or by means of the figure, we have

$$\left.\begin{array}{ll} \sin(90^\circ + y) = \cos y. & \cos(90^\circ + y) = -\sin y. \\ \tan(90^\circ + y) = -\cot y. & \cot(90^\circ + y) = -\tan y. \\ \sec(90^\circ + y) = -\csc y. & \csc(90^\circ + y) = \sec y. \end{array}\right\} (17)$$

EXERCISES.

1. What is the complement of 18°, 100°, 190°, − 40° ?
2. What is the supplement of 15°, 120°, 200°, 300°, − 50° ?
3. Construct the angle $90° + y$, when $y > 180°$ and $< 270°$.
4. Construct the angle $90° + y$, when $y > 270°$ and $< 360°$.
5. If $\sin x = \cos nx$, find x.

   [From (16) $\sin x = \cos (90° - x)$, which substituted in the example gives $\cos (90° - x) = \cos nx$; $\therefore\ 90° - x = nx$, and $x = \frac{90°}{1 + n}$.]
6. If $\cos x = \sin 2x$, find x.
7. If $\tan x = \cot nx$, find x.
8. Find x from the equation $\cot (90° - x) = \tan (45° - x)$.
9. Given $2x + y = \frac{1}{2}\pi$ and $\sin x = \cos 2y$, find x and y.
10. Given $\sin x = \cos 3y$ and $\tan 3x = \cot y$, to find x and y.
11. Given $\sin 3x = \cos 2y$ and $3x + 2y = 90°$, to find x and y.
12. Prove that $(a \cos \beta)^2 + (a \sin \beta \sin \gamma)^2 + (a \sin \beta \cos \gamma)^2 = a^2$.
13. What angle is that of which the cotangent equals twice the cosine?
14. Show that if $m^2 - n^2$ and $2mn$ be the lengths of the sides respectively of a right-angled triangle, the hypothenuse will be $m^2 + n^2$.

    [Substituting any pairs of numbers for m and n in the expressions in the preceding example will give respectively the sides and hypothenuse of a right triangle. Thus, if $m = 3$, and $n = 2$, the sides will be 5 and 12, and the hypothenuse 13.]
15. Find the relations between the functions for an angle of 45°.

    [Substitute $y = 45°$ in equations (16).]
16. Reduce equations (17) when $y = 45°$.
17. Reduce equations (16) when $y = 90°$.
18. Reduce equations (17) when $y = 90°$.
19. Show that $\sin (90° - \frac{1}{2}y) = \cos \frac{1}{2}y$.
20. $\tan (90° + \frac{1}{4}y) = ?$
21. $\cos (90° - ny) = ?$
22. Given $mx - ny = 90°$, and $\sin ax = \cos by$, to find x and y.

    [From the first of (16) and the second equation of the example, we have

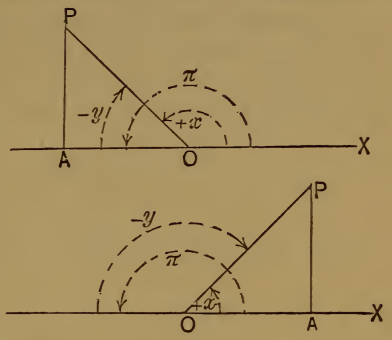
    $$\cos by = \sin (90° - by) = \sin ax;$$

    $$\therefore\ 90° - by = ax.$$

    Hence we find

    $$x = \frac{b + n}{an + bm} 90°,\ y = \frac{m - a}{an + bm} 90°.]$$

27. *Functions of* $180° - y$. In the first of equations (17), making $y = 90° - y_1$, we have for the first member $\sin x = \sin [90° + 90° - y_1)] = \sin (180° - y_1)$; and for the last member $\cos (90° - y_1) = \sin y_1$ [from equations (16)]; hence, dropping subscripts, we have.



$$\sin (180° - y) = \sin y.$$

Or, from the figure, we have

$$\sin x = \sin (180° - y) = AP \div OP = o \div r = \sin y,$$

and this relation is true whatever be the value of y.

By means of either of these methods applied to determining the other functions, we find

$$\left.\begin{array}{ll} \sin (180° - y) = \sin y. & \cos (180° - y) = -\cos y. \\ \tan (180° - y) = -\tan y. & \cot (180° - y) = -\cot y. \\ \sec (180° - y) = -\sec y. & \csc (180° - y) = \sec y. \end{array}\right\} (18)$$

The first of these is, *The sine of an angle equals the sine of its supplement.*

The second is, *The tangent of an angle equals* MINUS *the tangent of its supplement.*

EXERCISES.

1. Construct the angle $180° - y$, when $y > 180°$ and $< 270°$.
2. Construct the angle $180° - y$, when $y > 270°$ and $< 360°$.
3. If $\sin ny^2 = \sin (180° - y)$, find y.
4. If $\cos (90° + ny) = \cos (180° - my)$, find y.

5. Given $\tan(90° + ay) = \cot(180° - by)$, to find y.
   [In this example $y = 0$, or $a = b$.]
6. Is the relation $\tan(180° - y) = \cot(90° - y)$ possible?
7. $\sin(180° - \frac{1}{2}y) = ?$
8. $\cot(180° - \frac{1}{4}y) = ?$

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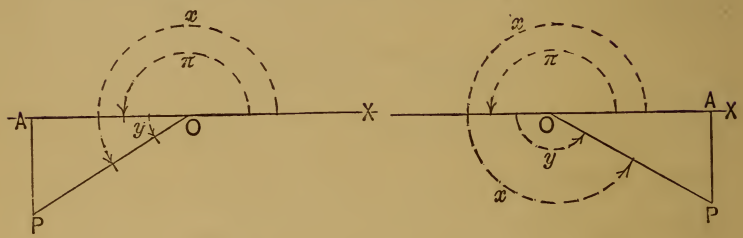
28. *Functions of* $(180° + y)$.



Substituting $-y$ for y in equations (18), or from the figure, we find

$$\left.\begin{array}{ll} \sin(180° + y) = -\sin y. & \cos(180° + y) = -\cos y. \\ \tan(180° + y) = \tan y. & \cot(180° + y) = \cot y. \\ \sec(180° + y) = -\sec y. & \csc(180° + y) = -\csc y. \end{array}\right\} (19)$$

29. *Functions of* $(270° \pm y)$.

Making $y = 90° - y$ in equations (19) and reducing by means of equations (16) and (17), we have

$$\left.\begin{array}{ll} \sin(270° \pm y) = -\cos y. & \cos(270° \pm y) = \pm \sin y. \\ \tan(270° \pm y) = \mp \cot y. & \cot(270° \pm y) = \mp \tan y. \\ \sec(270° \pm y) = \pm \csc y. & \csc(270° \pm y) = -\sec y. \end{array}\right\} (20)$$

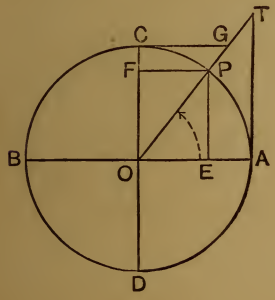
The functions of $n \cdot 360° \pm y$ are the same as for $\pm y$. It will be noticed that the functions of $90° \pm y$, $180° \pm y$. $270° \pm y$, may be expressed as functions of the angle y.

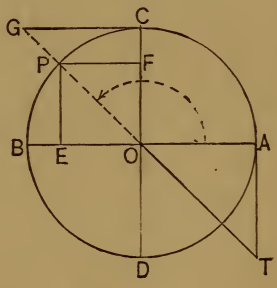
EXERCISES.

1. Construct $270° \pm y$ when $y > 0$ and $< 90°$.
2. Construct $270° \pm y$ when $y > 90 < 180°$.
3. Find the functions of $270° \pm y$ when $y = 45°$.
4. Construct $180° + y$ when $y = -30°$; also if $y = -135°$.
5. Show that $\sin^2(270° \pm y) + \cos^2(270° \pm y) = 1$.
6. Deduce equations (19) directly from the figure.
7. Deduce equations (20) directly from a figure.

TRIGONOMETRICAL LINES.

30. Making r unity in group (B), all the trigonometrical functions may be represented directly by right lines. With a radius $OA = OP = 1$, describe a circum-





ference, and AOP being the angle, draw EP and AT perpendicular to OA, and FP and CG parallel to OA, then

$$\sin AOP = \frac{EP}{OP} = \frac{EP}{1} = EP.$$

$$\cos AOP = \frac{OE}{OP} = \frac{OE}{1} = OE = FP.$$

$$\tan AOP = \frac{AT}{OA} = \frac{AT}{1} = AT.$$

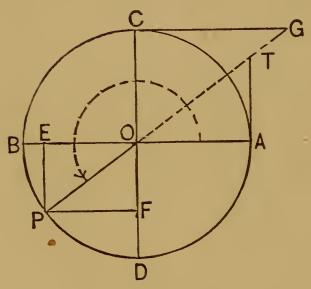
$$\cot AOP = \frac{OA}{AT} = \frac{CG}{OC} = \frac{CG}{1} = CG.$$

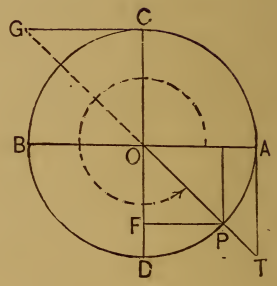
$$\sec AOP = \frac{OT}{OA} = \frac{OT}{1} = OT.$$

$$\csc AOP = \frac{OG}{OC} = OG.$$

$$\text{vrs } AOP = EA.$$

$$\text{cvs } AOP = FC.$$





These results stated as follows are *geometrical* definitions of the trigonometrical functions.

Let A be the origin of the arc, OA the initial radius, OP the terminal side of the angle, C, at a quadrant's distance from A, a secondary origin, and CD a secondary diameter; then in all the figures:

The sine of an arc (or *angle*) *is the perpendicular* FROM *the initial diameter to the terminus of the arc.*

Thus, EP is the sine of the arc AP, positive above BA and negative below.

The cosine of an arc (or *angle*) *is the perpendicular from the secondary diameter to the terminus of the arc.*

Thus, $FP = OE$ is the cosine of the arc AP, positive to the right of CD, and negative to the left.

The trigonometric tangent of an arc (or *angle*) *is that part of the geometric tangent at the origin of the arc limited by the diameter prolonged through the terminus of the arc.*

Thus, AT, drawn tangent to the arc at A, and limited by the diameter through P prolonged, is the trigonometrical tangent of AOP; positive above OA, and negative below.

The cotangent of an arc (or *angle*) *is that part of the geometric tangent at the secondary origin limited by the diameter through the terminus of the arc.*

Thus, CG drawn tangent to the arc at C, and limited by the diameter through P prolonged, is the cotangent of AOP; positive to the right of OC and negative to the left.

The secant of an arc (or *angle*) *is the distance from the centre of the arc to the extremity of the tangent.*

Thus, OT, the distance from the centre O to the extremity T of the tangent AT, is the secant of AOP; positive when the terminus P of the arc is on the secant, negative when P is on the extension of the secant, as in the second and third figures.

The cosecant of an arc (or *angle*) *is the distance from the centre of the arc to the extremity of the cotangent.*

Thus, OG, the distance from the centre O to the extremity G of the cotangent CG, is the cosecant of AOP;

positive when P is on the cosecant, and negative when on its extension.

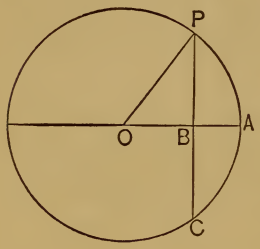
The versed sine of an arc (or *angle*) *is the distance from the foot of the sine to the origin of the arc.*

Thus, EA is the versed sine of the arc AP, and is positive for all real arcs.

The coversed sine of an arc (or *angle*) *is the distance from the foot of the cosine to the secondary origin.*

Thus, FC is the coversed sine of the arc AP, and is positive for all real arcs.

It should be remembered that these lines, in trigonometry, represent ratios—abstract numbers—and not linear quantities.



31. *The sine of an arc equals one-half the chord of double the arc.*

Thus, PB is the sine of the arc AP to the radius unity. But PB is one-half the chord PC of the arc PAC, which is twice the arc PA.

EXERCISES.

1. Find from a geometrical figure the limits of the trigonometrical functions, the values for which have been given on page 16.
2. Construct an angle whose cosine is $\frac{3}{4}$.
   [In the first figure of article 30 take $OE = \frac{3}{4}\ OA$, and erect the perpendicular EP; the radius OP will be the terminus of the required angle].
3. Construct the cosine of an angle whose cotangent is 2.
4. Construct all the trigonometrical functions of an angle whose tangent is -1 and whose sine is negative.

5. Show from a figure by means of similar triangles that $\tan x = \frac{\sin x}{\cos x}$.

Show that the following are true:

6. $\sin 405° = \sin 45°$.
7. $\sin 580° = -\sin 40°$.
8. $\sin 45° > \frac{1}{2} \sin 90°$.
9. $(\sin 60° + \sin 45°) > 1$.
10. $\sin (60° + 45°) < 1$.
11. $\frac{\sin 90°}{\cos 270°} = \infty$.
12. $\frac{\sin 180°}{\cos 90°}$ is indeterminate.
13. $\frac{\sin 360°}{\cos 180°} = 0$.

---

32. If $x = \sin y$, we may write as its equivalent $y = \sin^{-1} x$,* and read it "y equals the arc whose sine is x," or "y equals the angle whose sine is x." †

The several expressions

$$\sin^{-1} x, \quad \tan^{-1} x, \quad \cot^{-1} x, \quad \cos^{-1} x, \text{ \&c.},$$

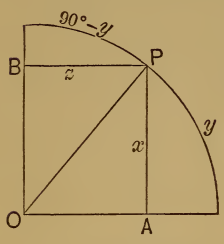
---

* The inverse functions are usually written thus: $\sin^{-1} x$, where 1 is used instead of the curve as in the text. This form is supposed to have been suggested by algebraic expressions like $y = m^{-1} x$; but m^{-1} is a reciprocal, whereas the inverse functions are not reciprocals. The old notation is unfortunate, and has rather been tolerated than approved; and although it is generally better to continue the use of an objectionable notation, especially when in common use, than to introduce a new one, yet, in this case, the modification proposed is so slight and the form is so suggestive of the real nature of the function, we trust it will be approved. The suggestion of its use was made to the author by Professor J. R. Paddock, of the Stevens High School.

† The former reading is sometimes preferred because the terms of the equation are at once strictly homogeneous as line functions. Thus, y being an arc, is linear, and so also is the sine as a geometrical line. The latter, however, is equally correct.

are called *inverse functions, circular functions,* or *anti-functions.*

We will have



$$\sin \sin^{-)} x = x, \quad \sin^{-)} \sin y = y \tag{21}$$

$$\sin \cos^{-)} z = x = \sqrt{1 - z^2}. \tag{22}$$

$$\sin^{-)} \cos y = \sin^{-)} z = 90° - y. \tag{23}$$

Or, more generally,

$$\sin^{-)} \sin y = n\pi + (-1)^n y, \tag{24}$$

where n is an integer, giving n values. But

$$\sin \sin^{-)} x$$

has but one value, and equals x.

33. *The reciprocal* may be indicated in the usual way, by a negative exponent, when no ambiguity results; otherwise a parenthesis should be employed.

Thus,

$$y = \sin \frac{1}{x} = \sin x^{-1}; \tag{25}$$

$$y = \sin^{-)} \frac{1}{x} = \sin^{-)} x^{-1}; \tag{26}$$

$$y = \frac{1}{\cos y} = (\cos y)^{-1} = \cos^{-1} y; \tag{27}$$

but the last form should not be used, for by most writers it designates an inverse function.

$$y = \frac{1}{\sin^{-)} x} = (\sin^{-)} x)^{-1} = \sin^{-)\,-1} x,$$

but the last form had better not be used for the reason just given.

$$y = \tan^{-)} \frac{1}{\cot \frac{1}{y}} = \tan^{-)} (\cot y^{-1})^{-1}.$$

EXERCISES.

1. If $y = \sin^{-)} \frac{2}{3}$, find $\cos y$, and construct a figure showing the several parts.
   [Taking the sine of both members we have $\sin y = \sin \sin^{-)} \frac{2}{3} = \frac{2}{3}$;
   $$\therefore \cos y = \sqrt{1 - \sin^2 y} = \tfrac{1}{3}\sqrt{5}.]$$
2. If $y = \tan^{-)} 3$, find $\sec y$.
3. If $\sin y = \sin^{-)} x$, find x in terms of y and y in terms of x.
   $$Ans.\ x = \sin \sin y, \quad y = \sin^{-)} \sin^{-)} x.$$
   [$\sin \sin y$ is not $\sin^2 y$, but is the sine of an arc whose length equals $\sin y$. Thus $\sin \frac{1}{6}\pi = \sin 30° = \frac{1}{2}$, and $\sin \frac{1}{2}$ is the sine of an arc whose length is one half the radius, or $\frac{1}{2} \times \frac{180°}{\pi} = 29\frac{1}{2}$ degrees nearly.]
4. By means of a figure or by analysis show that $\sin^{-)} x = \csc^{-)} \frac{1}{x}$
   $$= \pm \cos^{-)} \sqrt{1 - x^2} = \pm \sec^{-)} \frac{1}{\sqrt{1 - x^2}} = \pm \tan^{-)} \frac{x}{\sqrt{1 - x^2}}$$
   $$= \pm \cot^{-)} \frac{\sqrt{1 - x^2}}{x}.$$
5. Show that $\sin \tan^{-)} x = \frac{x}{\sqrt{1 + x^2}}$.
6. If $\tan \cos^{-)} x = \sec y$, show that $x = \cos \tan^{-)} \sec y$.
7. If $\sin \cos^{-)} \frac{1}{2} = \cos x$, find $\sin x$.
8. If $\sin^2 y^{\frac{1}{2}} = \cos 0°$, show that $y = \frac{1}{4}\pi^2$
9. If $\sin^{-)} y = \cos^{-)} x$, find y. $Ans.\ y = \sqrt{1 - x^2}$.
10. If $\sin \cos^{-)} \tan y = 0$, show that $y = \frac{1}{4}\pi$.

11. Show that $\sin 30° + \sin 45° > \sin (30° + 45°)$.
12. Show that $\sin 45° + \sin 45° > \sin 90°$.
13. Show that $2 \sin 30° > \sin 2 \times 30°$.
14. Show that $\cos 30° < 2 \cos 60°$.
15. Find the trigonometrical functions of 135°.

[Observe that $135° = 90° + 45°$, and substitute in equations (17). Or, $135° = 180° - 45$, and equations (18) will give the desired solution.]

16. Find the trigonometrical functions of 120°.
17. Given $\sec x = \left[\frac{4 \tan \sin^{-1} \frac{1}{2}}{3 \quad \tan \frac{1}{6}\pi}\right]^{\cos \frac{\pi}{3}}$, to find x.
18. From a geometrical figure similar to those on p. 27, show that the sine of an angle equals the sine of its supplement, or $\sin (180° - x) = \sin x$.
19. Show by means of the figures on page 28, that equations (19) and (20) are true.
20. Prove that $\sin (y - 90°) = -\cos y$.
21. Prove that $\cos (y - 90°) = \sin y$.
22. $\sin (y - 180°) = ?$
23. $\cos (y - 180°) = ?$

---

34. **Multiple angles.** If x be an angle, nx will be an angle n times x, and is called a multiple angle. The general properties of multiple angles will be discussed further on, but the functions of certain multiples of $\frac{1}{2}\pi$ may be readily found in this place.

Thus $\sin (4n + 1)\frac{\pi}{2} = 1$, for all integral values of n;

for the angle will be $(2n\pi + \frac{1}{2}\pi)$ where $2n\pi$ will be n complete circumferences, and $\frac{1}{2}\pi$ added will make n circumferences and 90° more; hence the sine will be the same as $\sin 90°$.

Generally, if n be an integer, either positive or negative, we have

$$\begin{matrix}\sin\\ \cos\end{matrix}\left[2\,n\,\frac{\pi}{2}+y\right]=(-1)^n\begin{bmatrix}\sin y\\ \cos y\end{bmatrix} \tag{28}$$

$$\begin{matrix}\sin\\ \cos\end{matrix}\left[(2\,n+1)\,\frac{\pi}{2}+y\right]=(-1)^n.\begin{bmatrix}\cos y\\ -\sin y\end{bmatrix} \tag{29}$$

Functions which repeat themselves at regular intervals are called *periodic functions.* Trigonometry may be defined as the algebra of periodic functions.

EXERCISES.

If n be an integer, including 0, show that:

1. $\sin 4n\frac{\pi}{2}=0$; $\sin(4n+2)\frac{\pi}{2}=0$.
2. $\sin(4n+3)\frac{\pi}{2}=-1$; $\cos(4n+2)\frac{1}{2}\pi=-1$.
3. $\tan\frac{1}{2}(2n\pi)=0$; $\cos(4n+1)\frac{1}{2}\pi=0$.
4. $\tan(2n+1)\frac{1}{2}\pi=\pm\infty$; $\cot\frac{1}{2}(2n\pi)=\mp\infty$.
5. $\sin(2n+1)\frac{1}{4}\pi=(+,+,-,-)\frac{1}{2}\sqrt{2}$.

---

35. *Impossible values.* The rules applicable to real functions may be extended to operations upon unreal or imaginary functions. Thus, $\sin x=2$ is impossible according to the definition of the sine, but considering the equation to be true in *fact* as it is in *form*, we have

$$\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-4}=\sqrt{-3}=\sqrt{3}\sqrt{-1}$$

$$\tan x=\frac{\sin x}{\cos x}=-\frac{2}{3}\sqrt{3}\sqrt{-1},$$

where the imaginary form shows that the original assumption was untrue.

EXERCISES.

1. If sec $x = \frac{1}{2}$, find expressions for all the other trigonometrical functions.
2. If tan $x = \sqrt{-1}$, find expressions for the other trigonometrical functions.
3. If cos $x = 2$, find the other trigonometrical functions.
4. If vsn $x = -1$, find the other functions.
5. Find tan x, given sec $x = \frac{1}{2}$ cos x.
6. Find cos x if sin $x = 2$ tan x.

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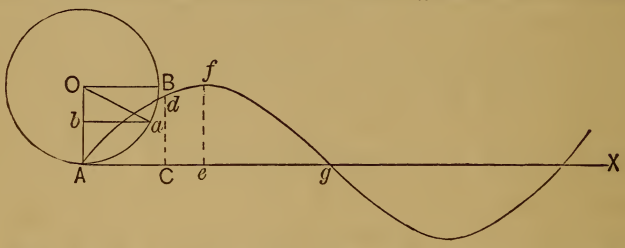
36. The sinusoid. If on the right line AX, there be laid off AC = the arc Aa, Ae = the quadrant AB, and so



on, and at C a perpendicular $Cd = ab$ = the sine of aOA; at e, the perpendicular $ef = BO$ = sin 90°, and so on, and a continuous curve be traced through the ends of the perpendiculars, the curve is called "the sinusoid." The coördinates of its points are expressed by the equation $y =$ sin x, and shows the relation between an arc and its sine. Ag equals a semi-circumference; and the sinusoid will cross the line AX at distances equal to multiples of a semi-circumference.

All the other trigonometrical functions may be represented in a similar way.

EXERCISES.

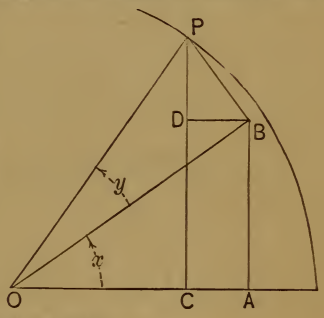
1. Construct a curve representing $y =$ cos x.
2. Construct $y =$ tan x.
3. Construct $y =$ sec x.
4. Construct $y =$ vsn x.

CHAPTER II.

FUNCTIONS OF TWO ANGLES.

37. *To find the sine of the sum and sine of the difference of two angles.*

Let AOB and BOP be two adjacent angles, then will their sum be AOP. From any point P in the terminal line OP, drop the perpendicular PC upon the initial line OA; also PB upon OB, BA upon OA, and BD upon PC. Since PB and PD are perpendicular respectively to the sides OB and OA of the angle AOB, the angle BPD will equal AOB.



Let $x = AOB = DPB$, $y = BOP$; then from the figure and the first of group (B) we have,

$$\sin(x+y) = \frac{CP}{OP} = \frac{CD+DP}{OP} = \frac{AB}{OP} + \frac{DP}{OP}.$$

But,

$$\frac{AB}{OP} = \frac{AB}{OB} \cdot \frac{OB}{OP} = \sin x \cos y,$$

and

$$\frac{DP}{OP} = \frac{DP}{BP} \cdot \frac{BP}{OP} = \cos x \sin y;$$

therefore, $\sin(x+y) = \sin x \cos y + \cos x \sin y.$ (30)

Writing $-y$ for y in this equation, gives

$$\begin{aligned}\sin(x-y) &= \sin x \cos(-y) + \cos x \sin(-y)\\ &= \sin x \cos y - \cos x \sin y. \qquad (31)\end{aligned}$$

Equations (30) and (31) show that—*The sine of the* SUM *of two angles equals the sine of the first into the cosine of the second* PLUS *the cosine of the first into the sine of the second.*

The sine of the DIFFERENCE of two angles equals the sine of the first into the cosine of the second MINUS the cosine of the first into the cosine of the second.

38. *To find the cosine of the sum and difference of two angles.*

From the preceding figure and group (*B*), we have,

$$\begin{aligned}\cos(x+y) &= \frac{OC}{OP} = \frac{OA - CA}{OP} = \frac{OA}{OP} - \frac{DB}{OP}\\ &= \frac{OA}{OB}\cdot\frac{OB}{OP} - \frac{DB}{PB}\cdot\frac{PB}{OP}\\ &= \cos x \cos y - \sin x \sin y. \qquad (32)\end{aligned}$$

Writing $-y$ for y, gives

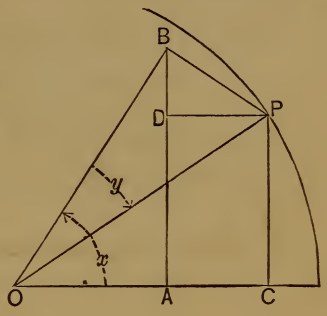
$$\cos(x-y) = \cos x \cos y + \sin x \sin y\,; \qquad (33)$$

that is—*The cosine of the* $\frac{\text{SUM}}{\text{DIFFERENCE}}$ *of two angles equals the product of their cosines* $\frac{\text{MINUS}}{\text{PLUS}}$ *the product of their sines.*

EXERCISES.

1. Represent geometrically the sine and cosine of x and y, and of $x - y$ in a single figure.
2. Deduce equation (31) from a figure.
3. Deduce equation (33) from a figure.
4. Deduce equation (32) from (30) by writing $90° - x$ for x and $-y$ for y.
5. Develop $\sin(a + b + c)$.

   [Let $a + b = x$, then develop $\sin(x + c)$, after which substitute the value of x and continue the development, finding

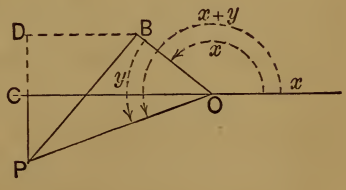


$$\sin(a + b + c) = \sin a \cos b \cos c + \cos a \sin b \cos c + \cos a \cos b \sin c - \sin a \sin b \sin c. \quad (34)$$

6. Show that

$$\cos(x + y + z) = \cos x \cos y \cos z - \sin x \sin y \cos z - \sin x \cos y \sin z - \cos x \sin y \sin z. \quad (35)$$

7. In exercise 6, make $z = 0$ and reduce.
8. In exercise 6 make z negative and reduce.
9. In exercises 5 and 6 make $x = y = z = a = b = c$, and find $\sin 3x$ and $\cos 3x$.
10. Find from a figure $\sin(x + y)$ when $x > 90° < 180°$, and $y > 0 < 90°$, and $x + y < 180°$.
11. Find $\cos(x + y)$ when $x > 90° < 180°$, $y < 90°$, and $x + y > 180°$.



12. Develop $\sin(3x - 2y)$, after which make $x = y$.
13. Given $\sin 45° = \cos 45° = \frac{1}{2}\sqrt{2}$, and $\sin 30° = \frac{1}{2}$, to find sine and cosine of 15°.

    [From equation (3), $\cos 30° = \sqrt{1 - \frac{1}{4}} = \frac{1}{2}\sqrt{3}$; and these several values in (31) and (33), will give $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$, and $\cos 15° = \frac{1}{4}(\sqrt{6} + \sqrt{2})$.]

14. Find sine and cosine of 75°.

[By means of (30) and (32); or by observing that 75° is the complement of 15°, we have $\sin 75° = \cos 15°$, and $\cos 75° = \sin 15°$.]

5. Show that $\sin(45° + y) = \frac{1}{2}\sqrt{2}(\sin y + \cos y)$.
6. Find the sine of $22\frac{1}{2}°$.

[Let $x = y = 22\frac{1}{2}°$ in (32), reducing by means of $\sin^2 22\frac{1}{2}° + \cos^2 22\frac{1}{2}° = 1$, and find $\sin 22\frac{1}{2}° = \frac{1}{2}\sqrt{2 - \sqrt{2}}$.]

17. Find the sine and cosine of $7\frac{1}{2}°$.
18. Develop $\sin(90° - x)$.
19. Develop $\sin^2(x + y)$.
20. Develop $\sin(x + y)^2$.

Prove the following statements:

21. $\cos 75° = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$.
22. $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$.
23. $\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$.
24. $\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$.
25. $\cos^4 \varphi - \sin^4 \varphi = \cos 2\varphi$.
26. $\cos(\varphi - 45°) + \sin(\varphi - 45°) = \sqrt{2} \sin \varphi$.
27. $\cos(\varphi - 45°) = \sin(\varphi + 45°)$.
28. $\sin n\varphi \cos \varphi + \cos n\varphi \sin \varphi = \sin(n + 1)\varphi$.
29. $\cos(n - 1)\varphi \cdot \cos \varphi - \sin(n - 1)\varphi \cdot \sin \varphi = \cos n\varphi$.
30. $\sin n\varphi \cos(n - 1)\varphi - \cos n\varphi \cdot \sin(n - 1)\varphi = \sin \varphi$.
31. $\sin 60° + \sin 30° = 2 \sin 45° \cos 15°$.

[Make $60° = 45° + 15°$, and $30° = 45° - 15°$, and develop.]

32. $\sin 3\varphi + \sin 5\varphi = 2 \sin 4\varphi \cos \varphi$.
33. $\cos \frac{1}{3}\pi - \cos \frac{1}{2}\pi = 2 \sin \frac{5}{12}\pi \cdot \sin \frac{1}{12}\pi$.
34. $\sin 2\varphi = 2 \sin \varphi \cos \varphi$.
35. $\cos(60° + \varphi) + \cos(60° - \varphi) = \cos \varphi$.
36. $\sin(\theta + \varphi) \cdot \cos \theta - \cos(\theta + \varphi) \sin \theta = \sin \varphi$.
37. $\sin \varphi = 2 \sin \frac{1}{2}\varphi \cos \frac{1}{2}\varphi$.
38. $\sin \frac{1}{2}\varphi = 2 \sin \frac{1}{4}\varphi \cos \frac{1}{4}\varphi$.
39. $\cos(\alpha + \beta) - \sin(\alpha - \beta) = 2 \sin(45° - \alpha) \cos(45° - \beta)$.
40. Solve the equation $a \sin \varphi + b \cos \varphi = c$.

[Let m and β be two auxiliary quantities determined by the relation

$$m \sin \beta = a.$$
$$m \cos \beta = b;$$

from which we find $\tan \beta = a \div b$, and $m = \sqrt{a^2 + b^2}$.

Substituting in the given equation gives

$$m \sin \beta \sin \varphi + m \cos \beta \cos \varphi = c,$$

or

$$m \cos (\beta - \varphi) = c;$$

$$\therefore \cos (\beta - \varphi) = \frac{c}{m} = \frac{c}{\sqrt{a^2 + b^2}},$$

and

$$\varphi = \beta - \cos^{-1} \frac{c}{\sqrt{a^2 + b^2}}.\Big]$$

41. Find r and φ from the equations

$$r \cos (\varphi + \alpha) = a,$$
$$r \sin (\varphi + \beta) = b.$$

[Expand, multiply the first by $\sin \beta$, the second by $\cos \alpha$, take the difference and find $r \sin \varphi$. Then multiply the first by $\cos \beta$, the second by $\sin \alpha$, add and find $r \cos \varphi$.]

Ans. $\varphi = \tan^{-1} \dfrac{b \cos \alpha - a \sin \beta}{b \sin \alpha + a \cos \beta}$;

$$r = \frac{b \cos \alpha - a \sin \beta}{\cos (\alpha - \beta) \sin \tan^{-1} \dfrac{b \cos \alpha - a \sin \beta}{b \sin \alpha + a \cos \beta}}.$$

42. Given

$$r \cos \varphi \cos \theta = -2,$$
$$r \cos \varphi \sin \theta = +3,$$
$$r \sin \beta \qquad = -4,$$

to find r, φ, θ.

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39. *Discussion of equations* (30), (31), (32), and (33). Let $x = y$ in (31) and (33), then

$$\sin 0^\circ = \sin x \cos x - \cos x \sin x = 0.$$
$$\cos 0^\circ = \cos x \cos x + \sin x \sin x = \cos^2 x + \sin^2 x.$$

But since x may be any angle, and $\cos 0^\circ$ has but one value, it follows that $\sin^2 x + \cos^2 x = a$ *constant*, and may be taken as unity. Hence, generally,

$$\sin 0^\circ = 0, \qquad \cos 0^\circ = 1.$$

Since 90° is the complement of 0°, we also have

$$\sin 90^\circ = \cos 0^\circ = 1,$$
$$\cos 90^\circ = \sin 0^\circ = 0.$$

Letting $x = y = 90^\circ$ in (30) and (32), we have

$$\sin 180^\circ = \sin 90^\circ \cos 90^\circ + \cos 90^\circ \sin 90^\circ$$
$$= 1 \times 0 + 0 \times 1 = 0,$$

and

$$\cos 180^\circ = \cos 90^\circ \cos 90^\circ - \sin 90^\circ \sin 90^\circ = -1.$$

Let $x = 90^\circ$, then

$$\sin (90^\circ + y) = \sin 90^\circ \cos y + \cos 90^\circ \sin y = \cos y,$$
$$\sin (90^\circ - y) = \sin 90^\circ \cos y - \cos 90^\circ \sin y = \cos y,$$
$$\cos (90^\circ + y) = \cos 90^\circ \cos y - \sin 90^\circ \sin y = -\sin y,$$
$$\cos (90^\circ - y) = \cos 90^\circ \cos y + \sin 90^\circ \sin y = \sin y;$$

the first and third of which have been previously deduced, equations (17), and the second and fourth are in equations (16).

Similarly, if $x = 180^\circ$, we find the functions of the sine and cosine of $180^\circ + x$ and $180^\circ - x$ as given in equations (18) and (19).

Similarly, the functions for $x = 270^\circ$, 360°, $n\frac{\pi}{2}$, $4n\pi$, and any other angles may be found by substituting in the same equations.

Let $x = 0$, then (31) gives

$$\sin(-y) = \sin 0^\circ \cos y - \cos 0^\circ \sin y = -\sin y;$$

and (33) gives

$$\cos(-y) = \cos 0^\circ \cos y + \sin 0^\circ \sin y = \cos y;$$

which are in equations (13) and (14).

Equation (30) may be considered as a *general equation* in this science in the sense that when combined with the definitions of group (*B*) all other equations of trigonometry may be deduced from it.

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40. *To find the tangent of the sum of two angles, and also of their difference.*

Dividing equation (30) by (32) gives

$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$$

$\div$ by $\cos x \cos y$,

$$= \frac{\dfrac{\sin x \cos y}{\cos x \cos y} + \dfrac{\cos x \sin y}{\cos x \cos y}}{1 - \dfrac{\sin x \sin y}{\cos x \cos y}}$$

reducing by means of equations (1) and (2), gives

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}; \qquad (36)$$

that is, *The tangent of the sum of two angles equals the sum of their tangents divided by* 1 MINUS *the product of their tangents.*

Making y negative and reducing by equations (14) gives

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \qquad (37)$$

41. *To find the cotangent of the sum and difference of two angles.*

Taking the reciprocals of the two preceding equations, we have according to equation (10)

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y} \tag{38}$$

$$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x} \tag{39}$$

FUNCTIONS OF DOUBLE THE ARC.

42. Making $x = y$ in (30), (32), (36), and (38), we have

$$\sin 2x = 2 \sin x \cos x \tag{40}$$

$$\cos 2x = \cos^2 x - \sin^2 x \tag{41}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \tag{42}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x} \tag{43}$$

FUNCTIONS OF THE ANGLE IN TERMS OF HALF ANGLE.

43. For x substitute $\frac{1}{2}x$ in (40), (41), (42), and (43), and we have

$$\sin x = 2 \sin \tfrac{1}{2}x \cos \tfrac{1}{2}x \tag{44}$$

$$\cos x = \cos^2 \tfrac{1}{2}x - \sin^2 \tfrac{1}{2}x \tag{45}$$

$$= 1 - 2 \sin^2 \tfrac{1}{2}x \tag{46}$$

$$= 2 \cos^2 \tfrac{1}{2}x - 1 \tag{47}$$

$$\tan x = \frac{2 \tan \frac{1}{2}x}{1 - \tan^2 \frac{1}{2}x}. \tag{48}$$

$$\cot x = \frac{\cot^2 \frac{1}{2}x - 1}{2 \cot \frac{1}{2}x} \tag{49}$$

FUNCTIONS OF ONE-HALF AN ANGLE.

44. From (46) we have

$$\sin \tfrac{1}{2}x = \sqrt{\tfrac{1}{2}(1 - \cos x)} \tag{50}$$

from (47),

$$\cos \tfrac{1}{2}x = \sqrt{\tfrac{1}{2}(1 + \cos x)} \tag{51}$$

(50) ÷ (51),

$$\tan \tfrac{1}{2}x = \sqrt{\frac{1 - \cos x}{1 + \cos x}} \tag{52}$$

(51) ÷ (50),

$$\cot \tfrac{1}{2}x = \sqrt{\frac{1 + \cos x}{1 - \cos x}} \tag{53}$$

(44) ÷ (47),

$$\tan \tfrac{1}{2}x = \frac{\sin x}{1 + \cos x} \tag{54}$$

(44) ÷ (46),

$$\cot \tfrac{1}{2}x = \frac{\sin x}{1 - \cos x} \tag{55}$$

reciprocal (55),

$$\tan \tfrac{1}{2}x = \frac{1 - \cos x}{\sin x} \tag{56}$$

reciprocal (54),

$$\cot \tfrac{1}{2}x = \frac{1 + \cos x}{\sin x} \tag{57}$$

45. From (30), (31), (32), and (33) we obtain by addition and subtraction,

$$\sin(x+y)+\sin(x-y)=2\sin x\cos y \tag{58}$$
$$\sin(x+y)-\sin(x-y)=2\cos x\sin y \tag{59}$$
$$\cos(x+y)+\cos(x-y)=2\cos x\cos y \tag{60}$$
$$\cos(x+y)-\cos(x-y)=-2\sin x\sin y \tag{61}$$

Let $x+y=s$ and $x-y=d$, then $x=\frac{1}{2}(s+d)$, and $y=\frac{1}{2}(s-d)$, which substituted above gives

$$\sin s+\sin d=2\sin\tfrac{1}{2}(s+d)\cos\tfrac{1}{2}(s-d) \tag{62}$$
$$\sin s-\sin d=2\cos\tfrac{1}{2}(s+d)\sin\tfrac{1}{2}(s-d) \tag{63}$$
$$\cos s+\cos d=2\cos\tfrac{1}{2}(s+d)\cos\tfrac{1}{2}(s-d) \tag{64}$$
$$\cos s-\cos d=-2\sin\tfrac{1}{2}(s+d)\sin\tfrac{1}{2}(s-d) \tag{65}$$

which forms are convenient for logarithmic computation.

---

(62) ÷ (63),

$$\frac{\sin s+\sin d}{\sin s-\sin d}=\frac{\tan\frac{1}{2}(s+d)}{\tan\frac{1}{2}(s-d)} \tag{66}$$

(62) ÷ (64),

$$\frac{\sin s+\sin d}{\cos s+\cos d}=\tan\tfrac{1}{2}(s+d) \tag{67}$$

(63) ÷ (64),

$$\frac{\sin s-\sin d}{\cos s+\cos d}=\tan\tfrac{1}{2}(s-d) \tag{68}$$

(62) ÷ (65),

$$\frac{\sin s+\sin d}{\cos s-\cos d}=-\cot\tfrac{1}{2}(s-d) \tag{69}$$

Formula (66) gives,—*The sum of the sines of two angles is to their difference as the tangent of one-half the sum of the angles is to the tangent of one-half their difference.*

EXERCISES.

1. $\tan^{-1}\frac{5}{7} + \tan^{-1}\frac{1}{6} = \frac{1}{4}\pi$.
   [Take tangent of the left member according to equation (36), $\tan(\frac{1}{4}\pi) = 1$.]
2. $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{1}{4}\pi$.
3. $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{2}{3} = \frac{1}{4}\pi$.
4. $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\dfrac{a+b}{1-ab}$.
5. $\sin x = \dfrac{2\tan\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}$.
   [Substitute, $\tan\frac{1}{2}x = \dfrac{\sin\frac{1}{2}x}{\cos\frac{1}{2}x}$.]
6. $\cos x = \dfrac{1-\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}$.
7. $\tan\frac{1}{2}x = \dfrac{1+\sin x-\cos x}{1+\sin x+\cos x}$.
8. $\tan(45^\circ \pm y) = \cot(45^\circ \mp y) = \dfrac{\cos y \pm \sin y}{\cos y \mp \sin y}$.
9. $\sin mx = 2\sin(m-1)x\cos x - \sin(m-2)x$.
   [In (58) make $x = (m-1)y$. Another value for $\sin mx$ may be found by making $x = (m-1)y$ in (59).]

   $\therefore \sin 2x = 2\sin x\cos x$

   $\sin 3x = 2\sin 2x\cos x - \sin x = 4\sin x\cos^2 x - \sin x$

   $\sin 4x = 2\sin 3x\cos x - \sin 2x = 8\sin x\cos^3 x - 4\sin x\cos x$.
10. $\cos mx = 2\cos(m-1)x\cos x - \cos(m-2)x$.
    [Deduced from (60). Another value may be found from (61).]

    $\therefore \cos 2x = 2\cos^2 x - 1 = -2\sin^2 x + 1$

    $\cos 3x = 2\cos 2x\cos x - \cos x = 4\cos^3 x - 3\cos x$

    $\cos 4x = 2\cos 3x\cos x - \cos 2x = 8\cos^4 x - 8\cos^2 x + 1$.

11. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$.

[Develop $\tan(x + 2x)$ by eq. (36), or make $x = y = z$ in exercise 12.]

12. $\tan(x + y + z) = \dfrac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan x \tan z - \tan y \tan z}$.

13. Given $\sin x = e^{\sqrt{-1}}$ find $\cos x$.

[$e = 2.718281824 +$, the base of the Naperian system of logarithms.]

14. Given

$$\cos x = \frac{1}{2}\left(e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}\right) \qquad (a)$$

$$-\sin x = \frac{1}{2}\left(e^{x\sqrt{-1}} - e^{-x\sqrt{-1}}\right)\sqrt{-1} \qquad (b)$$

to find $\cos^2 x + \sin^2 x = 1$.

[Equations (a) and (b) are known as *Euler's formulæ*, having been discovered by the celebrated mathematician LEONHARD EULER. They are proved by means of higher analysis, but are placed here for convenience of reference, and to afford an exercise in the use of *imaginaries*. Since they are true for all values of x, they illustrate the fact that *imaginaries* may be so combined as to represent *real* numerical values. One of the most interesting of these simple relations is shown in the mystical combinations given in the next exercise.]

15. In equation (a) of the preceding example, make $x = \frac{1}{2}\pi$ and $i = \sqrt{-1}$, and show that

$$i^{-i} = i^{\frac{1}{i}} = \sqrt[i]{i} = \left[\sqrt{-1}\right]^{\frac{1}{\sqrt{-1}}} = \sqrt{e^{\pi}} = 4.8095 +.$$

16. In De Moivre's formula, which is

$$(\cos\varphi + \sqrt{-1}\sin\varphi)^n = \cos n\varphi + \sqrt{-1}\sin n\varphi,$$

substitute $\varphi = \frac{1}{2}\pi$, and $n = 1, 2, 3$, etc., and reduce, finding the n^{th} powers of $\sqrt{-1}$.

17. By means of equations (*a*) and (*b*), exercise 14, show that

$$\sqrt{-1}\tan x = \frac{e^{2x\sqrt{-1}} - 1}{e^{2x\sqrt{-1}} + 1}.$$

18. Generalizing De Moivre's formula, exercise 16, gives

$$(\cos\varphi + \sqrt{-1}\sin\varphi)^n = \cos n(2r\pi + \varphi) + \sqrt{-1}\sin n(2r\pi + \varphi), \quad (c)$$

where $r = 0, 1, 2, 3$, etc., and n a fraction. By means of this equation the several roots of unity may be found.

What are the three roots of unity? Make $\varphi = 0$, $n = \frac{1}{3}$, and $r = 0, 1, 2$, in eq. (*c*).

Ans. $1, \frac{1}{2}(-1 + \sqrt{-3}), \frac{1}{2}(-1 - \sqrt{-3})$.

19. If $A + B + C = 180°$, we may find

$$\cos A + \cos B + \cos C = 1 + 4\sin\tfrac{1}{2}A \sin\tfrac{1}{2}B \sin\tfrac{1}{2}C$$
$$\sin A + \sin B + \sin C = 4\cos\tfrac{1}{2}A \cos\tfrac{1}{2}B \cos\tfrac{1}{2}C$$
$$\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$$
$$\cos 2A + \cos 2B + \cos 2C = -4\cos A \cos B \cos C - 1$$
$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$
$$\cot\tfrac{1}{2}A + \cot\tfrac{1}{2}B + \cot\tfrac{1}{2}C = \cot\tfrac{1}{2}A \cot\tfrac{1}{2}B \cot\tfrac{1}{2}C$$
$$\cos\tfrac{1}{2}A + \cos\tfrac{1}{2}B + \cos\tfrac{1}{2}C = 4\cos(45° + \tfrac{1}{4}A)\cos(45° + \tfrac{1}{4}B)\cos(45° + \tfrac{1}{4}C)$$

The following series are readily proved by means of higher mathematics.

20. $$\sin x = \frac{x}{\underline{1}} - \frac{x^3}{\underline{3}} + \frac{x^5}{\underline{5}} - \frac{x^7}{\underline{7}} + \cdots$$

21. $$\cos x = 1 - \frac{x^2}{\underline{2}} + \frac{x^4}{\underline{4}} - \frac{x^6}{\underline{6}} + \cdots$$

22. $$\sin^{-1} y = y + \tfrac{1}{2}\frac{y^3}{3} + \frac{1 \cdot 3}{2 \cdot 4}\cdot\frac{y^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\cdot\frac{y^7}{7} + \cdots$$

23. $$\sin x = x\left(1 - \frac{x^2}{1^2\pi^2}\right)\left(1 - \frac{x^2}{2^2\pi^2}\right)\left(1 - \frac{x^2}{3^2\pi^2}\right) \cdots\cdots$$

24. $$\cos x = \left(1 - \frac{2^2x^2}{1^2\pi^2}\right)\left(1 - \frac{2^2x^2}{3^2\pi^2}\right)\left(1 - \frac{2^2x^2}{5^2\pi^2}\right) \cdots\cdots$$

25. $$\frac{\pi}{2} = 1 + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{5} + \text{etc.}$$

[By making $y = 1$ in 22d example.]

26. $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \text{etc.}$

27. $\frac{\pi}{4} = 1 - \frac{1}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{5}\left(\frac{1}{2}\right)^5 - \text{etc.}$
$+ \left[\frac{1}{3} - \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 + \text{etc.}\right].$

28. $\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \ldots$
which is known as Willis' expression.

29. $\frac{1}{8}\pi^2 = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + .$ (Todhunter).

30. $\frac{1}{4}\pi^2 = 1 + \frac{1}{3} + \frac{4^2}{3.5.6} + \frac{4^2 . 6^2}{3.5.6.7.8} + \text{etc.}$

31. $(\cos x)^n = 1 - \frac{nx^2}{\lfloor 2} + \frac{n(3n-2)x^4}{\lfloor 4} - \frac{n[15(n-1)^2+1]x^6}{\lfloor 6} + \ldots$

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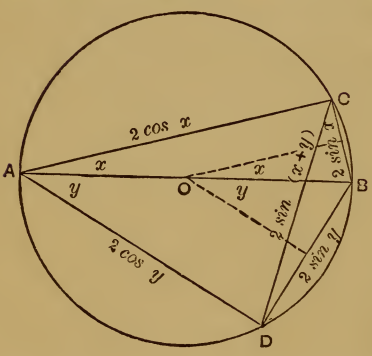
46. The following methods will afford some variety in deducing certain expressions.



Let AB be the diameter of a circle whose radius is unity, $BAC = x$, $BAD = y$. Draw CD CB, and BD. Then,

$AC = 2\cos x$, $AD = 2\cos y$,
$CB = 2\sin x$, $DB = 2\sin y$,
$CD = 2\sin(x+y)$,

since the sine of $x + y$ is half the chord of the arc CD. The product of the diagonals of an inscribed quadrilateral equals the sum of the products of its opposite sides, therefore

$$AB \,.\, CD = CB \,.\, AD + AC \,.\, DB$$

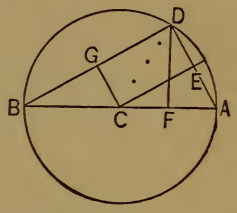
or,

$$2 \times 2\sin(x+y) = 4\sin x \cos y + 4\cos x \sin y$$

which, dropping the common factor 4, gives equation (30).

Constructing a figure with the angles x and y both on the same side of the diameter, the theorem for $\sin(x - y)$ may be as quickly proved in a similar manner.

In the annexed figure let the arc AD, or angle $ACD = x$; draw DB, DA, $CE \perp DA$, $CG \perp BD$; then



$$ACE = ABD = ADF = \tfrac{1}{2}x,$$

$$AE = \sin \tfrac{1}{2}x,\ CE = DG = \tfrac{1}{2}DB = \cos \tfrac{1}{2}x,$$

$$DF = \sin x,\ CF = \cos x,\ BF = 1 + \cos x,\ FA = 1 - \cos x\ ;$$

and

$$\cos \tfrac{1}{2}x = \frac{BF}{BD} = \frac{1 + \cos x}{2 \cos \tfrac{1}{2}x};$$

reducing,

$$2 \cos^2 \tfrac{1}{2}x = 1 + \cos x$$

which is equation (51).

Again

$$\sin \tfrac{1}{2}x = \frac{DF}{BD} = \frac{\sin x}{2 \cos \tfrac{1}{2}x};$$

$$\therefore \sin x = 2 \sin \tfrac{1}{2}x \cos \tfrac{1}{2}x$$

which is equation (44).

Again,

$$\tan \tfrac{1}{2}x = \frac{DF}{BF} = \frac{\sin x}{1 + \cos x}$$

which is equation (51).

Again,

$$\tan \tfrac{1}{2}x = \frac{FA}{DF} = \frac{1 - \cos x}{\sin x}$$

which is equation (56).

Multiplying together the last two equations gives

$$\tan \tfrac{1}{2}x = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

which is equation (52).

Again,

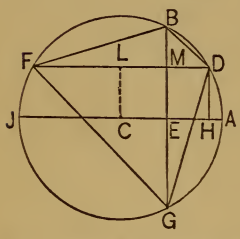
$$\sin \tfrac{1}{2}x = \frac{FA}{DA} = \frac{1 - \cos x}{2 \sin \tfrac{1}{2}x}$$

reducing,

$$\sin^2 \tfrac{1}{2}x = \tfrac{1}{2}(1 - \cos x)$$

which is equation (50).

47. Now constructing another figure, describe a circle with C as a centre, take $AD = y$, $AB = x = AG$; draw FD parallel to the diameter AJ, BEG, and DH perpendicular to AJ, also, FB, BD, DG and FG. Then will the arc $DB = x - y$, the chord



$$BD = 2 \sin \tfrac{1}{2}(x - y), \qquad DH = \sin y$$
$$DG = 2 \sin \tfrac{1}{2}(x + y), \qquad CH = \cos y$$
$$BE = \sin x, \qquad CE = \cos x$$
$$MG = GE + ME = \sin x + \sin y$$

then, since BGD is an angle in the circumference and measured by $\frac{1}{2}BD$,

$$\sin DGM, \text{ or } \sin \tfrac{1}{2}(x - y) = \frac{MD}{GD} = \frac{EH}{GD} = \frac{\cos y - \cos x}{2 \sin \tfrac{1}{2}(x + y)},$$

which gives equation (65).

Again,

$$\tan \tfrac{1}{2}(x - y) = \frac{MD}{MG} = \frac{\cos y - \cos x}{\sin x + \sin y},$$

which is the reciprocal of (69).

Again,

$$\tan DFG = \frac{MG}{FM} = \frac{EG + ME}{LM + FL} = \frac{EG + ME}{LM + LD}$$

or

$$\tan \tfrac{1}{2}(x + y) = \frac{\sin x + \sin y}{\cos x + \cos y}$$

which is equation (67).

Again,

$$\tan \tfrac{1}{2}(x - y) = \frac{BM}{FM} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

which is equation (68).

Again,

$$\frac{\tan \tfrac{1}{2}(x + y)}{\tan \tfrac{1}{2}(x - y)} = \frac{MG}{MB} = \frac{\sin x + \sin y}{\sin x - \sin y},$$

which is equation (66).

We have

$$FGB = 90° - GFM = 90° - \tfrac{1}{2}(x + y);$$
$$\therefore FB = 2 \sin (FGB) = 2 \cos \tfrac{1}{2}(x + y).$$

Then

$$\sin BFD, \text{ or } \sin \tfrac{1}{2}(x - y) = \frac{BM}{FB} = \frac{\sin x - \sin y}{2 \cos \tfrac{1}{2}(x + y)}$$

which gives equation (63).

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48. *Construction of tables.* Since the student is not expected to make trigonometrical tables, we will only state briefly how they may be made. To make a table of sines, find the length of arc for the required angle, the radius being unity, and substitute in the second member of example 20, page 49, and reduce. Thus, to find the sine of 10°, the arc will be $x = \frac{\pi}{180} \times 10 = 0.1745329$, which substituted in example 20, above referred to, and reduced will give $\sin 10° = 0.17365 +$. In this, or in some other way, the sine of all angles differing by 1′, and in some cases by 1″, have been computed and entered in a table. Then $\cos x = \sqrt{1 - \sin^2 x}$ gives the cosines, tan = sin ÷ cos gives the tangents, and cot = 1 ÷ tan, the cotangents. A table containing these values, is called a table of *natural* trigonometrical functions. The term *natural* is used to distinguish them from logarithmic functions.

A table of logarithmic trigonometrical functions consists of the logarithms of the functions increased by 10. Thus, $\sin 10° = 0.17365$, and $\log \sin 10° = \log 0.17365 = \bar{1}.239670$. Adding 10, we have 9.239670, and this number is entered in the table as log sin 10°. The number 10 is added to avoid negative characteristics. For directions for using the tables, see description just preceding the tables used.

CHAPTER III.

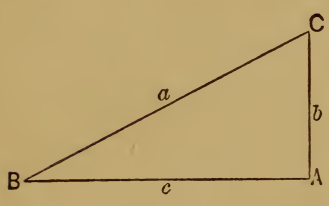
SOLUTION OF PLANE TRIANGLES.

49. **Right angled triangles.** The formulas in group (*A*) are directly applicable to the solution of right triangles. The parts of a triangle are its sides and angles.

EXAMPLES.

a. Solutions by means of Natural Functions.

1. In the right triangle BAC, if the angle $B = 43° 25' 18''$, and the hypothenuse $BC = 235$, find the remaining parts.



To find C. Since the three angles of a triangle $= 180°$, and $A = 90°$, therefore $B + C = 90°$, and $C = 90° - B$; hence C is the complement of B; and $C = 90° - 43° 25' 18'' = 46° 34' 42''$.

To find BA. From group (*A*) we have

$$\cos B = \frac{\textit{side adjacent}}{\textit{hypothenuse}} = \frac{BA}{BC} = \frac{c}{a};$$

By means of a table of natural cosines, we find $\cos 43° 25' = 0.72637$. To find it for $18''$ more, we assume that the function varies directly as

the angle for $1'$ — which, though not exact, is sufficiently so for ordinary practice. From the same table, $\cos 43^\circ 26' = 0.72617$, hence, $0.72637 - 0.72617 = 0.00020$ is the decrease in the cosine for $1' = 60''$; hence for $18''$ the decrease will be $\frac{18}{60}$ of $0.00020 = 0.00006$, and this subtracted from the $\cos 43^\circ 25'$, gives

$$\cos 43^\circ 25' 18'' = 0.72631;$$
$$\therefore BA = 235 \times 0.72631 = 170.68 +$$

To find the natural sine, the difference for the seconds must be *added* to the sine of the degrees and minutes of the smaller angle.

To find AC. From group (*A*) we have

$$\sin B = \frac{AC}{BC} = \frac{b}{a};$$
$$\therefore AC = a \sin B = 235 \times \sin 43^\circ 25' 18''$$
$$= 235 \times 0.68736 = 161.5296.$$

Collecting results, we have

Ans. $C = 46^\circ 34' 42''$
$BA = 170.68$
$AC = 161.5296.$

[It is generally advisable to compute all the known parts from the given ones, so that if an error is made in determining one of the parts it will not affect the computation of the other parts.]

Check. As a *check* upon the work some of the computed parts may be determined by involving some of the other computed parts. Thus, having found the angle C and the side b, we have

$$c = a \sin C = b \tan C.$$

Also

$$a^2 = b^2 + c^2.$$

2. In the right angled triangle BAC the angle B is $33° 30'$ and the side AC is 52.21. Find the other parts.

Ans. $BC = 94.6$; $AB = 78.88$; $C = 56° 30'$.

3. In the right angled triangle BAC the hypothenuse BC is 127.9 and the side CA is 97.72, to find the other parts.

Ans. $B = 49° 49' 15''$; $BA = 82.51$; $C = 40° 10' 45''$.

4. In the right angled triangle DEF, given the side EF 75, and the side DE 50.59, to find the other parts.

5. Given one of the acute angles $44°$, and the hypothenuse 405, to find the other parts.

6. In a right angled triangle one of the acute angles is $67° 0' 25''$ and the side opposite is 710, to find the other parts.

b. Solution by Means of Logarithmic Functions.

7. Given $a = 672.3412$, $B = 35° 16' 25''$, to find the other parts.

To find b. We have from group (A),

$$\sin B = \frac{b}{a};$$

$$\therefore b = a \sin B;$$

$$\therefore \log b = \log a + \log \sin B.$$

Add the logarithmic sine of B as found from the table of logarithmic functions, to the logarithm

of a, then will the number corresponding to the sum of the logarithms be the required number. The work may be arranged thus:

$a = 672.3412 \ldots\ldots\ldots \log 2.827590$
$B = 35° 16' 25'' \ldots\ldots \log \sin 9.761538$
$\therefore\ b = 388.264 \ldots\ldots\ldots\ldots \log 2.589128$

Ten has been dropped from the characteristic because the tabular logarithmic sine is ten too large. Finally, 388.264 is the number corresponding to the logarithm 2.589128.

To find c. We have $\cos B = \frac{c}{a}$;

$\therefore \log c = \log a + \log \cos B.$
$a = 672.3412 \ldots\ldots\ldots \log 2.827590$
$B = 35° 16' 25'' \ldots\ldots \log \cos 9.911905$
$c = 548.902 \ldots\ldots\ldots\ldots \log 2.739495$

To find C. We have $C = 90 - 35° 16' 25'' = 54° 43' 35''$.

Ans. $\begin{cases} b = 388.264 \\ c = 548.902 \\ C = 54° 43' 35'' \end{cases}$

Check. To test the accuracy of the work, compute the same quantity by different formulas.

Thus, we have $c = a \sin C$;
$\therefore \log c = \log a + \log \sin C$;
or
$\tan B = \frac{b}{c}$;
$\therefore \log \tan B = \log b - \log c.$

If the last formula be used, employing the computed values of b and c, and the value thus found for B agrees with that given in the example, it is very certain that not only are the values of b and c correct, but also the process of verification. They will be absolutely correct if there are no errors balancing each other.

8. Given the hypothenuse 65.07 and one of the acute angles 39° 38′ 28″, to find the other parts.

Ans. 41.51; 50.11; 50° 21′ 32″.

9. Given the hypothenuse 2195 and acute angle 27° 38′ 50″, to find the other parts.

10. Given the hypothenuse 365 and one side 86, to find the other parts.

Ans. 13° 37′ 41″; 76° 22′ 19″; 354.724.

11. Given the hypothenuse 0.897 and one of the sides 0.00086, to find the other parts.

Ans. 3′ 17″.7; 89° 56′ 42″.3; 0.89538.

[To find an angle less than 2° from the logarithmic table, see EXPLANATION OF THE TABLES.]

12. Given the hypothenuse 672.3 and one of the sides 548.9, to find the other parts.

Ans. 35° 17′; 54° 43′; 388.2.

13. In a right angled triangle given one of the sides 7643.5 and the angle opposite 37° 18′, to find the other parts.

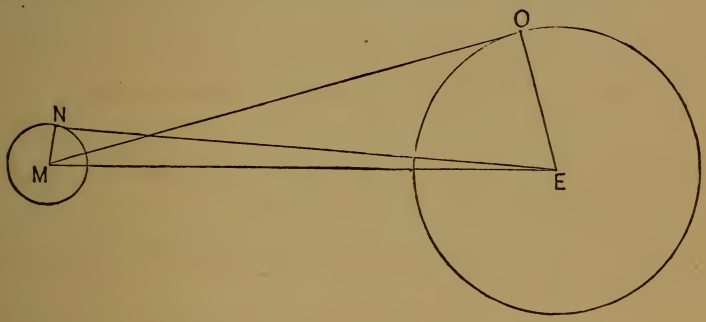
14. Given the two sides 1728 and 1575, to find the other parts.

15. Given the two sides 246.32 and 380.07, to find the other parts.

16. In the figure if M represents the moon and EO the earth, the radius of the earth $EO = 3956.2$ miles,

and the angle $OME = 57'$, required the distance ME, O being a right angle.

$ME = 238{,}614$ miles.



17. If EN is a tangent to the moon's disc at N, the angle NEM will be the moon's apparent semi-diameter. The apparent diameter of the moon being $31'\ 20''$, and the distance from the earth as found in the last example, what is the diameter of the moon?

Ans. 2174.8 miles.

18. A tower 150 feet high throws a shadow 75 feet long upon the horizontal plane of its base; what is the angle of elevation of the sun?

19. A railroad track makes a vertical rise of 150 feet in a distance of 3,000 feet, by uniform grade. What is the angle of the grade?

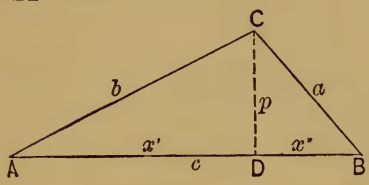
20. A rope whose length is 109 feet is fastened to the top of a tower 60 feet high, the lower end being carried as far as possible from the foot of the tower and fastened to the ground, required the angle which it makes with the horizon.

21. Given the earth's equatorial radius 3962.8 miles, and the sun's horizontal parallax 8″.87, find the distance of the earth from the sun.

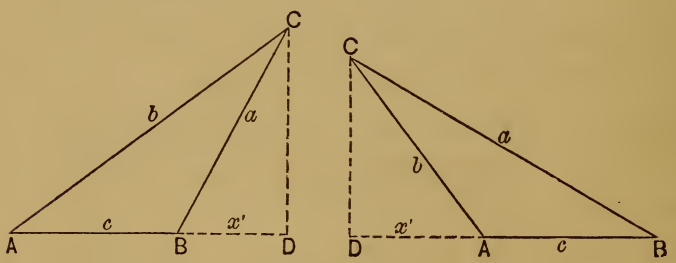
[The sun's horizontal parallax is what the apparent radius of the earth would appear to be if viewed from the centre of the sun.] *Ans.* 92,152,000 miles.

OF OBLIQUE ANGLED PLANE TRIANGLES.

50. **A general relation.** *The ratio of any two sides of a plane triangle equals the ratio of the sines of the angles opposite.*



Let ABC be any plane triangle the angles of which are denoted by A, B, C, and the sides opposite by a, b, c, respectively. From any angle C let fall the perpendicular p upon the opposite side, or that side prolonged if necessary.



From the right triangle ADC we have

$$\sin A = \frac{p}{b},$$

and from BDC

$$\sin B = \frac{p}{a},$$

and dividing the former by the latter, cancelling p, we have

$$\frac{\sin A}{\sin B} = \frac{a}{b}. \tag{70}$$

Similarly,

$$\frac{\sin A}{\sin C} = \frac{a}{c}, \tag{71}$$

which was to be proved.

Generally,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}. \tag{72}$$

The common value of this ratio is sometimes called *the modulus* of the triangle.

---

51. **Case I.** *Given two angles and one side.*
Let b be the side, A and B the angles.

Then

$$C = 180^\circ - (A + B), \tag{73}$$

and (71),

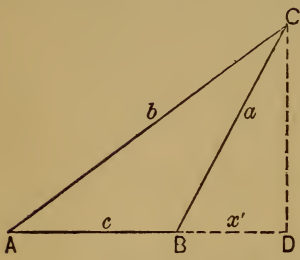
$$a = c\frac{\sin A}{\sin C}, \tag{74}$$

and (72),

$$c = b\frac{\sin C}{\sin B} \tag{75}$$

EXAMPLES.

1. In the triangle ABC, let $A = 48° 3'$, $C = 40° 14'$, and and $AC = 376$, to find the other parts,



From (73)

$$B = 91° 43'$$

To find c, use (75), as follows :

$B = 91° 43'$ ar. co.	log sin 0.000195
$C = 40° 14'$	log sin 9.810167
$b = 376$	log 2.575188
$\therefore\ c = 243$	log 2.385550

(The log sin 91° 43′ is the same as log cos 1° 43′.)

To find $BC = a$, use (74), and find $a = 279.8$.

Ans. $B = 91° 43'$, $a = 279.8$, $c = 243$.

2. In the triangle ABC, let $A = 10° 12'$, $B = 96° 36'$, and $BC = 5.55$, to find the other parts.

3. In the triangle ABC, let $A = 81° 30' 10''$, $B = 40° 30' 44''$ and $AB = 696$, to find the other parts.

4. In the triangle ABC, let $A = 68° 4' 20''$, $B = 56° 32', 35''$, and $AC = 696.75$, to find the other parts.

---

52. **Case II.** *Given two sides and angle opposite one of them.*

Let a, b, B, be the given parts.

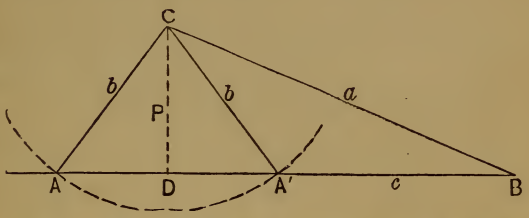
To find A. From (72) we have

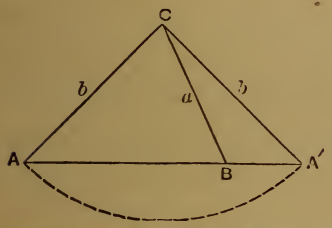
$$\sin A = \frac{a}{b} \sin B. \qquad (76)$$

But since the angle A and its supplement have the same sine, there will generally be two angles found by (76) which will fulfil the conditions of the problem, and

hence two triangles, BAC and $BA'C$, and the result is said to be *ambiguous*. The several conditions involved in this case are most readily discussed geometrically.

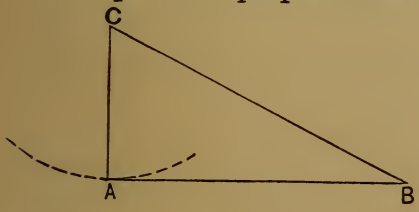
Thus, to construct the triangle, at B draw the lines BC and BA making $ABC = B$. On BC take the distance a, and with C as a centre and radius $CA = b$ describe an arc AA', and if it cuts BA in two points on the same side of B there will be two triangles.

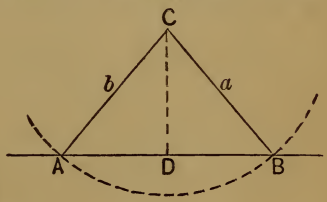




If $b > a$ there will be only one triangle. For if B is acute the triangle will be ABC, but if B is obtuse the triangle will be $BA'C$. Both triangles are not admissible for the same data.

If b equals the perpendicular from C to BA, and B is

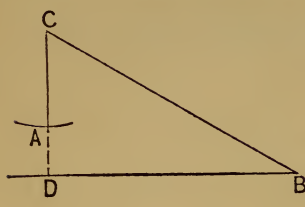




acute, there will be one triangle. But if B be obtuse there will be no triangle.

If $b = a$ and B is acute, there will be one triangle, ABC. But if $b = a$ and B is obtuse, or right, there will be no triangle, for b and a will coincide.

If B is acute, right, or obtuse, and b is less than the perpendicular from C to BD, there will be no triangle.



If there be two triangles, represent the two angles found by A' and A'', and the two sides by c' and c''.

EXAMPLES.

1. Given $a = 31.238$, $b = 49$, and $A = 32° 18'$, to find B, C, and c.

To find B. Eq. (76)

$a = 31.238$ ..... ar co log 8.505316
$b = 49$ ..... log 1.690196
$A = 32° 18'$ ..... log sin 9.727828

$\therefore B' = 56° 56' 57''$ .. log sin 9.923340
$B'' = 123° 3' 3''$

To find C′ and C″.

$C' = 180° - (32° 18' + 56° 56' 57'') = 90° 45' 3''$
$C'' = 180° - (32° 18' + 123° 3' 3'') = 24° 38' 57''$

To find c′ and c″.

$C' = 90° 45' 3''$ ..... log sin 9.999996
$C'' = 24° 38' 57''$ ..... log sin 9.620196
$A = 32° 18'$ " ar co log sin 0.272172 ..... 0.272172
$a = 31.238$ ..... log 1.494693 ..... 1.494693

$\therefore c' = 58.460$ ..... log 1.766862
$c'' = 24.381$ 1.387062

Ans. $\begin{cases} B' = 56° 56' 57'' \\ C' = 90° 45' 3'' \\ c' = 58.460 \end{cases}$ or $\begin{cases} B'' = 123° 3' 3'' \\ C'' = 24° 38' 57'' \\ c'' = 24.381 \end{cases}$

2. In the triangle ABC, let $BC = 94.26$, $AC = 126.2$ and $A = 27° 50'$, to find the other parts.

$B = 38° 52' 46''$. $C = 113° 17' 14''$. $c = 185.439$.
or $141° 7' 14''$. $11° 2' 46''$. 39.682.

3. Given $a = 91.06$, $b = 77.04$, $A = 51° 9' 6''$. Find the other parts.

$B = 41° 13'$; $C = 87° 37' 54''$; $c = 116.82$.

4. Given $a = 1257.5$, $c = 1751$, $A = 31° 17' 19''$. Find the other parts.

5. Given $a = 40$, $b = 55$, $A = 60°$. *Ans. Impossible.*

6. Given $a = 43$, $b = 45$, $A = 120°$. *Ans. Impossible.*

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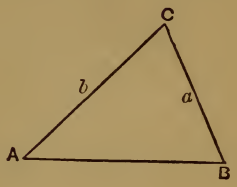
53. **Case III.** *Given two sides and the included angle.*

Let a, b, C, be the given parts.



From (70) we have

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

Adding one to both members, we have by reduction,

$$\frac{a+b}{b} = \frac{\sin A + \sin B}{\sin B}.$$

Similarly, subtracting 1 from both members,

$$\frac{a-b}{b} = \frac{\sin A - \sin B}{\sin B}.$$

Dividing gives,

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B}.$$

This reduced by (66) gives

$$\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}; \tag{77}$$

that is, *The sum of two sides of a plane triangle is to their difference, as the tangent of half the sum of the angles opposite is to the tangent of half their difference.*

To find $A + B$ *we have*

$$A + B = 180^\circ - C. \tag{78}$$

To find $A - B$ *we have from* (77) *and* (78)

$$A - B = 2 \tan^{-1}\left[\frac{a-b}{a+b} \tfrac{1}{2} \tan (180^\circ - C)\right] \tag{79}$$

Reducing the quantity in the brackets, [ ], and finding the angle corresponding to this tangent by means of a table of trigonometrical functions, and multiplying the result by 2 will give $A - B$.

To find A *and* B. We have from (78) and (79)

$$A = \tfrac{1}{2}[(A+B) + (A-B)] \tag{80}$$

$$B = \tfrac{1}{2}[(A+B) - (A-B)] \tag{81}$$

To find c. We have from (71)

$$c = \frac{\sin C}{\sin A} a \tag{82}$$

A solution may also be made by means of right triangles, as will be shown in article 55.

EXAMPLES.

1. In a plane triangle ABC, given $a = 748$, $b = 375$, $C = 63^\circ\ 35'\ 30''$, to find the other parts of the triangle.

$a + b = 1123$ a. c. log 6.949620
$a - b = 373$ log 2.571709
$\frac{1}{2}(A + B) = 58° 12' 15''$ log tan 10.207660
$\therefore \frac{1}{2}(A - B) = 28° 10' 54''$ log tan 9.728989
$A = 86° 23' 9''$
$B = 30° 1' 21''$ ..... a. c. log sin 0.300744
$C = 63° 35' 30''$ ..... log sin 9.952137
$b = 375$ ..... log 2.574031
$c = 671.29$ ..... log 2.826912

Ans. $A = 86° 23' 9''$, $B = 30° 1' 21''$, $c = 671.29$.

2. Given $c = 304$, $a = 280.3$ $B = 100°$, to find the other parts.
Ans. $A = 38° 3' 33''$, $C = 41° 56' 57''$, $b = 447.86$.

3. Given $a = 468$, $b = 365$, $C = 82° 22' 22''$, to find the other parts.

4. Given $a = .3756$, $b = .2375$, $C = 68° 25'$, to find the other parts.

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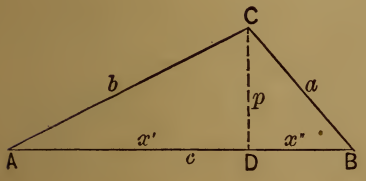
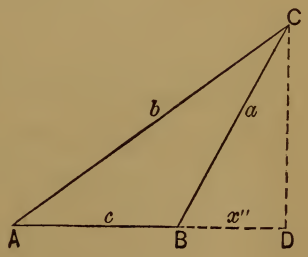
54. **Case IV.** *Given the three sides of a triangle, to find the angles.*





From the figure we have

$$AD = x' = b \cos A, \quad DB = x'' = a \cos B,$$

$$\left.\begin{aligned} \therefore\ x' + x'' &= c = b\cos A + a\cos B \\ \text{similarly,}\quad b &= a\cos C + c\cos A \\ a &= c\cos B + b\cos C \end{aligned}\right\} \tag{83}$$

In the second figure $\cos B$ is negative; hence x'' will be negative, and we have

$$\begin{aligned} c = x' + (-x'') &= x' - x'' = b\cos A - a(-\cos B) \\ &= b\cos A + a\cos B \end{aligned}$$

as before.

From the 1st of (83),

$$a\cos B = c - b\cos A$$

squaring, $a^2\cos^2 B = c^2 - 2bc\cos A + b^2\cos^2 A$,

from (72) $a^2\sin^2 B = b^2\sin^2 A$,

adding,

$$\left.\begin{aligned} a^2 &= c^2 + b^2 - 2bc\cos A \\ \text{similarly,}\quad b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}\right\} \tag{84}$$

From (84) we have

$$\left.\begin{aligned} \cos A &= \frac{-a^2 + b^2 + c^2}{2bc} \\ \cos B &= \frac{a^2 - b^2 + c^2}{2ac} \\ \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}\right\} \tag{85}$$

Equations (85) will solve this case by means of natural functions. To adapt them to logarithmic computation, they may be transformed thus:

Subtracting each member of the first of (85) from unity we have

$$\begin{aligned} 1 - \cos A &= \frac{2bc + a^2 - b^2 - c^2}{2bc} = \frac{a^2 - (b-c)^2}{2bc} \\ &= \frac{(a + b - c)(a - b + c)}{2bc}. \end{aligned}$$

But $1 - \cos A = 2 \sin^2 \frac{1}{2}A$ (eq. (50)); and letting $2s = (a + b + c)$, we have $a + b - c = 2(s - c)$, and $a - b + c = 2(s - b)$; hence

$$\sin^2 \tfrac{1}{2}A = \frac{(s-b)(s-c)}{bc};$$

$$\left.\begin{aligned} \therefore \sin \tfrac{1}{2}A &= \sqrt{\frac{(s-b)(s-c)}{bc}} \\ \text{similarly,} \quad \sin \tfrac{1}{2}B &= \sqrt{\frac{(s-a)(s-c)}{ac}} \\ \sin \tfrac{1}{2}C &= \sqrt{\frac{(s-a)(s-b)}{ab}} \end{aligned}\right\} \quad (86)$$

Adding each member of (85) to unity, and reducing in a similar manner, we find

$$\left.\begin{aligned} \cos \tfrac{1}{2}A &= \sqrt{\frac{s(s-a)}{bc}} \\ \cos \tfrac{1}{2}B &= \sqrt{\frac{s(s-b)}{ac}} \\ \cos \tfrac{1}{2}C &= \sqrt{\frac{s(s-c)}{ab}} \end{aligned}\right\} \quad (87)$$

Dividing each of equations (86) by the corresponding one of (87), gives

$$\left.\begin{aligned} \tan \tfrac{1}{2}A &= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \\ \tan \tfrac{1}{2}B &= \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} \\ \tan \tfrac{1}{2}C &= \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} \end{aligned}\right\} \quad (88)$$

EXAMPLES.

1. In a plane triangle, given $a = 30$, $b = 25$, and $c = 20$, to find the angles.

By natural functions. The first of equations (85) gives

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{625 + 400 - 900}{1000} = 0.125,$$

hence

$$A = 82^\circ\ 49'\ 9''.$$

In a similar manner the values of B and C may be found.

2. In a plane triangle $a = 21.35$, $b = 12.17$, $c = 10.08$, to find the angles.

By logarithmic functions. Using equations (88),

		log		log.		log
$s = 21.80$	a. c.	8.661544	a. c.	8.661544	a. c.	8.661544
$s - a = 0.45$	a. c.	10.346787		$\bar{1}$.653213		$\bar{1}$.653213
$s - b = 9.63$		0.983626	a. c.	9.016374		0.983626
$s - c = 11.72$		1.068928		1.068928	a. c.	8.931072
		2)1.060885				
$\frac{1}{2}A = 73^\circ\ 34'\ 25''$....	tan	0.530442		2)18.400059		
$\frac{1}{2}B = 9^\circ\ 0'\ 22.''5$.....	tan	9.200029				2)$\bar{2}$.229455
$\frac{1}{2}C = 7^\circ\ 25'\ 12.''5$.....	tan	9.114727				

Ans. $A = 147^\circ\ 8'\ 50''$ $B = 18^\circ\ 0'\ 45''$ $C = 14^\circ\ 50'\ 25''$.

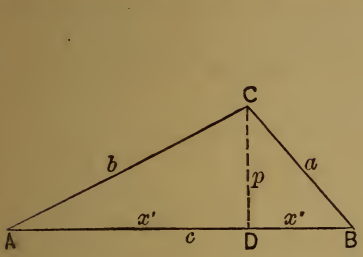
3. Given $a = 14$, $b = 17$, $c = 19$, to find the angles.

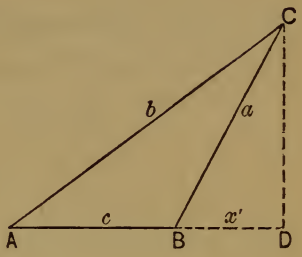
4. Given $a = 0.0237$, $b = 0.164$, $c = 0.0843$, to find the angles.

---

55. *Solution by means of right triangles.* Let ABC be any plane triangle, CD a perpendicular from any angle

C upon the opposite side AB. We may indicate briefly the solutions for the several cases.





Case I. *Given one side and two angles.*

Let $AC = b$, be the side, A and C the angles, then

$$B = 180° - (A + C); \quad BCD = 90° - B;$$
$$ACD = 90° - A, \qquad (89)$$

$$AD = b \cos A = x'; \quad CD = b \sin A = p;$$
$$CB = CD \div \cos BCD = a; \qquad (90)$$

$$DB = a \cos B = x''; \quad \therefore AB = x' + x''; \qquad (91)$$

observing that x'' in the second figure will be negative, since cos B will be negative.

Case II. *Given two sides and an angle opposite one of them.*

Let a and b be the sides and A the angle. Find AD and CD as in Case I. Then sin $B = p \div a$; $C = 180° - (A + B)$; $BD = a \cos B$; hence all the parts will be found.

Since B generally has two values in the equation sin $B = p \div a$, there will generally be two solutions.

Case III. *Given two sides and the included angle.*

Let the given parts be b, c, A. Find $AD = x'$, and $CD = p$ as in Case I. Then $DB = c - x' = x''$; tan $B = p \div x''$; $DCB = 90° - B$; $ACD = 90° - A$

$\therefore ACB = 90° - (A + B)$, and all the parts become known.

Case IV. *Given the three sides.*

Let $CD = p$, $AD = x'$, $BD = x''$; then

$$p^2 = b^2 - x'^2 = a^2 - x''^2,$$

and

$$x' + x'' = c,$$

These equations give

$$x' = \frac{-a^2 + b^2 + c^2}{2c} \qquad x'' = \frac{a^2 - b^2 + c^2}{2c} \tag{92}$$

Then

$$\cos A = \frac{x'}{b} \qquad \cos B = \frac{x''}{a}; \tag{93}$$

in which if x' or x'' be negative, the corresponding angle will be obtuse. The angle C may be found by dropping a perpendicular from A or B and proceeding as before; or, simply, $C = 180° - (A + B)$.

EXAMPLES.

1. Given $a = 250$ $A = 10° 12$ $B = 46° 36'$
   find $C = 123° 12'$ $c = 1181.30$ $b = 1025.74$
2. Given $a = 17.432$ $b = 19\,574$ $A = 38° 44' 12''$
   find $c' = 27.908$ $B' = 44° 1' 28''$ $C' = 97° 44' 30''$
   $c'' = 2.8404$ $B'' = 135° 58' 32''$ $C'' = 5° 47' 16''$
3. Given $a = 95.98$ $b = 66.28$ $C = 175° 19' 10''$
   find $A = 2° 46' 8''$ $B = 1° 54' 42''$ $c = 162.128$
4. Given $a = 6$, $b = 8$, $c = 10$; find the angles.

5. Given $a = 15$, $b = 15$, $c = 17$; find the angles.
6. Given $a = \sqrt{-2}$, $b = \sqrt{-3}$, $c = \sqrt{-7}$; find the angles.
7. Given $a = 3\sqrt{-1}$, $b = 4\sqrt{-1}$, $B = 60°\ 38'$, to find the other parts.

---

56. **Small angles.** For tables which give the values of the trigonometrical functions to minutes only, the student is obliged to compute the values for the seconds. This is usually done by assuming that the function varies directly as the angle from minute to minute. But for angles less than two degrees and near 90°, this is not sufficiently exact; hence these angles should be avoided, if possible, unless special tables are provided, which either give the values of the functions to seconds, or which give the means of computing the values of the small angles more accurately. In this work, the latter plan is adopted, as may be seen by consulting the first two pages of the table of logarithmic trigonometrical functions.

In this case the values of

$$\log \frac{\sin x}{x} = \log \sin x - \log x = q - l \qquad (94)$$

$$\log \frac{\tan x}{x} = \log \tan x - \log x = q - l \qquad (95)$$

are computed for each minute, and change slowly, as may be seen by examining the table, where even the sixth figure, in some cases, is the same for several *minutes* of arc. If the angle be given in seconds, we have

$$\sin x = x'' \frac{\sin x}{x}; \tag{96}$$

$$\therefore \log \sin x = \log x'' + q - l, \tag{97}$$

or $$\log x'' = \log \sin x - (q - l) \tag{98}$$

Similarly, $$\log \tan x = \log x'' + (q - l);$$

$$\therefore \log x'' = \log \tan x - (q - l), \tag{99}$$

where $q - l$ in the latter case is taken from the column next to that for tangents.

EXAMPLE. Given the base of a right triangle 4500 and altitude 5, to find the angles.

We have

$$\tan A = \frac{5}{4500}; \quad \therefore \log \tan A = \log 5 - \log 4500, \text{ or}$$

$\log 5 =$	0.698970
$\log 4500 =$	3.653213
difference,	$\bar{3}.045757$
adding	10
gives	7.045757
subtract $q - l$	4.685575
$\therefore x'' = 229''.1$	$\log$ 2.360182,

which is correct to 0.1 of a second. The other angle will be 89° 56′ 10″.9.

Having found 7.045757, we find the logarithm corresponding most nearly to it in the column having "Tang" at its head, and in the fourth column we find 4.685575 for the value of $q - l$, which being subtracted gives the logarithm of the seconds; the number of which is found in the table of the logarithms of numbers. For further instruction see EXPLANATION OF THE TABLES.

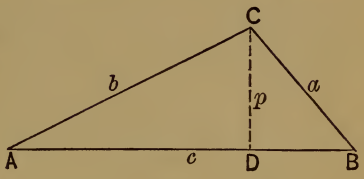
AREA OF TRIANGLES.

57. The area of a triangle equals one-half the product of the base by the altitude, or

$$K = \tfrac{1}{2}cp \tag{100}$$

where c = the base, p = the distance of the base from the opposite angle, and K = the area.

a. Given two sides and the included angle, or b, c, A.



Here

$$p = b \sin A,$$

$$\therefore K = \tfrac{1}{2}bc \sin A. \quad (101)$$

b. Given two sides and an angle opposite one of them, or a, b, B.

$$\sin A = \frac{a}{b} \sin B; \quad (102)$$

$$\therefore A = \sin^{-1}\left(\frac{a}{b} \sin B\right). \quad (103)$$

Then

$$p'' = a \sin C = a \sin (A + B) = a \sin \left[\sin^{-1}\left(\frac{a}{b} \sin B\right) + B\right]; \quad (104)$$

$$\therefore Area = \tfrac{1}{2}ab \sin \left[\sin^{-1}\left(\frac{a}{b} \sin B\right) + B\right]. \quad (105)$$

Expanding the sine of the quantity in the brackets by equation (30) and reducing, gives

$$Area = \tfrac{1}{2}a \sin B \left[a \cos B + \sqrt{b^2 - a^2 \sin^2 B}\right]. \quad (106)$$

Interpreting this result by means of the figure, we find that if a perpendicular be dropped from C upon the side c, dividing the latter into two segments, the term $a \cos B$ in the brackets will be the length of one segment, and the other term, $\sqrt{b^2 - a^2 \sin^2 B}$, will be the length of the other segment, and hence their sum will be the length of the unknown side c, and the factor $a \sin B$ will be the length of the perpendicular from C upon the unknown side c.

If a, b, A, be given, the solution will be of the same form and the result may be found by writing A for B in equation (106) and interchanging a and b.

c. Given two angles and the included side, or A, B, c.

First find a from equation (71),

$$\frac{a}{c} = \frac{\sin A}{\sin C} = \frac{\sin A}{\sin (A + B)};$$

$$\therefore a = c \frac{\sin A}{\sin (A + B)}; \quad (107)$$

then, $$p = a \sin B$$

substituting, $$= c \frac{\sin A \sin B}{\sin (A + B)}; \tag{108}$$

$$\therefore \text{Area} = \tfrac{1}{2} c^2 \frac{\sin A \sin B}{\sin (A + B)}. \tag{109}$$

d. Given the three sides, as a, b, c.
Let $AD = x$, then $DB = c - x$,

and $$p^2 = b^2 - x^2 = a^2 - (c - x)^2; \tag{110}$$

$$\therefore x = \frac{-a^2 + b^2 + c^2}{2c}, \tag{111}$$

which substituted above gives

$$p^2 = b^2 - \left(\frac{-a^2 + b^2 + c^2}{2c}\right)^2 = \frac{4b^2 c^2 - (-a^2 + b^2 + c^2)^2}{4c^2}$$

which being the difference of two squares reduces to

$$p^2 = \frac{(2bc - a^2 + b^2 + c^2)(2bc + a^2 - b^2 - c^2)}{4c^2}$$

$$= \frac{[-a^2 + (b + c)^2][a^2 - (b - c)^2]}{4c^2}$$

$$= \frac{(a + b + c)(-a + b + c)(a - b + c)(a + b - c)}{4c^2}$$

$$= \frac{4s(s - a)(s - b)(s - c)}{c^2};$$

$$\therefore p = \frac{2}{c}\sqrt{s(s - a)(s - b)(s - c)} \tag{112}$$

where $s = \frac{1}{2}(a + b + c)$. The area will be

$$K = \sqrt{s(s - a)(s - b)(s - c)}. \tag{113}$$

Or thus; $$CD = p = b \sin A$$

Eq. (44), $$= 2b \sin \tfrac{1}{2}A \cos \tfrac{1}{2}A$$

Eqs. (86), (87), $$= \frac{2}{c}\sqrt{s(s - a)(s - b)(s - c)};$$

Eq. (100), $\therefore K = \sqrt{s(s-a)(s-b)(s-c)}$,

as before.

If p' be the perpendicular from the angle A to the side a we may write, from inspection of (112)

$$p' = \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}; \tag{114}$$

and similar for the perpendicular from B to b,

$$p'' = \frac{2}{b}\sqrt{s(s-a)(s-b)(s-c)}; \tag{115}$$

and hence,

$$\frac{1}{p} + \frac{1}{p'} + \frac{1}{p''} = \frac{s}{K} \tag{116}$$

58. **Inscribed circle.** If a circle be inscribed in a triangle, and the centre joined with each of the angles by a right line, it will be divided into three triangles each having an altitude equal to the radius of the circle. Hence the area of the triangle will be

$$K = \tfrac{1}{2}ra + \tfrac{1}{2}rb + \tfrac{1}{2}rc = rs; \tag{117}$$

$$\therefore r = \frac{K}{s} = \frac{1}{s}\sqrt{s(s-a)(s-b)(s-c)} = \frac{pa}{2s} \tag{118}$$

59. **Circumscribed circle.** If R be the radius of the circumscribed circle, we will have

$$R = \frac{\frac{1}{2}a}{\sin A} = \frac{\frac{1}{2}b}{\sin B} = \frac{\frac{1}{2}c}{\sin C}. \tag{119}$$

60. **Escribed circles.** If r', r'', r''', be the radii of the escribed circles, opposite respectively to the sides a, b, c, then we have,

$$r' = \frac{K}{s-a}, \qquad r'' = \frac{K}{s-b}, \qquad r''' = \frac{K}{s-c} \tag{120}$$

$$\frac{1}{r} = \frac{1}{r'} + \frac{1}{r''} + \frac{1}{r'''} \tag{121}$$

$$K^2 = r \cdot r' \cdot r'' \cdot r''' \tag{122}$$

(Escribed circles are exterior to the triangle, tangent to one side and to the other two prolonged.)

EXERCISES.

Find the area of the triangles, the three perpendiculars, the radii of the inscribed, circumscribed, and escribed circles, given

1. $a = 12.5$, $b = 15.2$, $c = 20.5$.

Ans. $K = 94.641+$, $R = 10.288+$, $r = 3.927+$.
$p = 15.144+$, $p' = 12.452+$, $p'' = 9.223+$.
$r' = 8.156+$, $r'' = 10.633+$. $r''' = 26.289+$.

2. $a = 16.2$, $b = 22.2$, $C = 30° 20'$.

Ans. $K = 90.814+$, $R = 11.316+$ $r = 3.632+$.
$p = 11.211+$, $p' = 8.181+$ $p''' = 15.617+$.
$r' = 10.319+$, $r'' = 32.433+$ $r''' = 6.777+$.

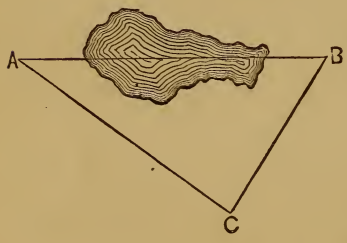
3. $a = 12.5$, $B = 40° 40'$, $C = 60° 10'$.

4. $A = 100° 10'$ $B = 50° 25'$ $c = 20.5$.

PRACTICAL EXAMPLES.

1. Find the area of a regular pentagon of which the sides are 12 feet each.

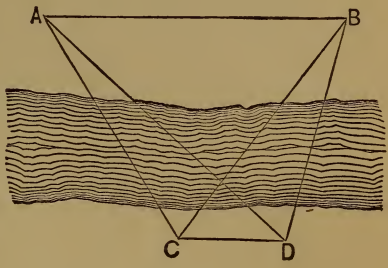
2. Find the area of a regular hexagon of which each side is 12 feet.



3. To determine the unmeasured distance AB, having given $AC = 235$ feet, $CB = 317$ feet, and the angle $ACB = 48° 45' 12''$.

4. From the top of a lighthouse 100 feet high, the angle of depression of a ship lying at anchor was $7° 43' 43''$, from the

foot of the lighthouse the angle of depression was 6° 12′ 13″; required the height of the hill on which the lighthouse was situated and the horizontal distance to the ship.



5. To determine the inaccessible distance AB, having given

$CD = 225$ feet, $ACB = 74° 15'$,
$BCD = 44° 27'$, $BDA = 66° 54'$,
and $CDA = 39° 38'$.

Ans. 528.265.

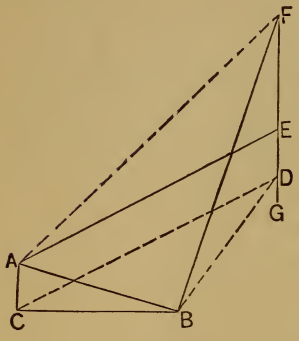
6. A headland was observed from a ship to bear directly east; after sailing S.W. 25 miles the same headland bore E. by N.; required the distance of the headland from the two points of observation.

[Instead of dividing the quadrant into 90 degrees, navigators divide it into eight equal parts called points, which are subdivided into quarters. The points as named on the card of the mariner's compass in the quadrant between north and east are as follows:

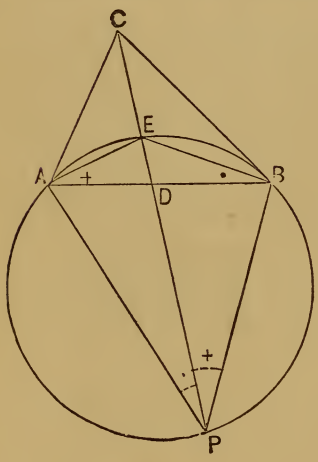
N., *N.* by *E.*, *N.N.E.*, *N.E.* by *N.*, *N.E.*, *N.E.* by *E.* *E.N.E.*, *E.* by *N.*, *E.*, and similarly for the other quadrants. Thus, *N.* by *E.* will be 11¼° *E.* of *N.*; *N.N.E.*, 22½° *E.* of *N.*, and so on.]

7. At one station a cloud bears N.N.W. and its angle of elevation is 53° 22′; at a second station, whose bearing from the first is N. by E., and distance one mile, the cloud bears west; required the height of the cloud. *Ans.* 1.427 *m.*

8. How far from the foot of a light-house can a light 350 feet high be seen out at sea, the radius of the earth being 3,956 miles? *Ans.* 22.901 *m.*



9. To determine the height of the pole FD, the base $AB = 250$ feet was measured, the end A of which was $AC = 12$ feet higher than the end B. When the transit was at B, the point D, where the horizontal line BD cut the pole was noted, the vertical angle $DBF = 12° 24'$, and the horizontal angle $DBC = 35° 15'$ were measured; and at A the horizontal angle $= DCB = 27° 51'$. Required the height FD. As a check upon the work, at A the point E may be noted, and the vertical angle EAF measured, by means of which EF may be determined, which added to ED should equal DF.



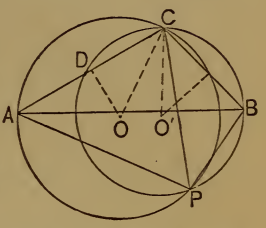
10. From a point P were measured the angles $APC = 33° 45'$, and $CPB = 22° 30'$. The distances $AC = 600$, $AB = 800$, $BC = 400$, being known, it is required to find AP and BP.

Draw a circle through the points A, B, P; join C and P, and from E, where CP intersects the circle, draw EA and EB; then will $EAB = CPB$, and $ABE = APC$, since the

former angles are subtended by the arc EB, and the latter by AE.

To find the point P by construction.

On one side of AC as a chord describe a segment of a circle which will contain an inscribed angle $= 33° 45'$, which may be done by making $ACO = 90° - 33° 45' = 56° 15'$, and intersecting CO by the perpendicular DO drawn through the middle of AC; the intersection O will be the centre of the arc, and OC the radius. Similarly, on the chord BC describe another segment of a circle that will contain an angle of $22° 30'$. The intersection P, of the two arcs will be the point sought. But four such arcs may be described, two having their centres on opposite sides of AC, and two others having their centres on opposite sides of BC; and the corresponding arcs will generally intersect in four points which may be designated by P_1, P_2, P_3, P_4. There can be no more than four points, but there may be less. In this problem there are four, as follows :



$$\begin{cases} AP_1 = 710.2 \\ BP_1 = 934\ 3 \\ CP_1 = 1042.5 \end{cases} \quad \begin{cases} AP_2 = 910.3 \\ BP_2 = 764.9 \\ CP_2 = 434.0 \end{cases} \quad \begin{cases} AP_3 = 164.3 \\ BP_3 = 960.5 \\ CP_3 = 729.6 \end{cases} \quad \begin{cases} AP_4 = 981.2 \\ BP_4 = 185.5 \\ CP_4 = 565.1 \end{cases}$$

11. From one station the angle of elevation of the top of a tower was observed to be $75° 18'$, from another station 400 feet farther from the tower the angle of elevation was $28° 45'$; required the height of the tower, the two stations being in the same horizontal plane.

SPHERICAL TRIGONOMETRY.

GEOMETRICAL PRINCIPLES.

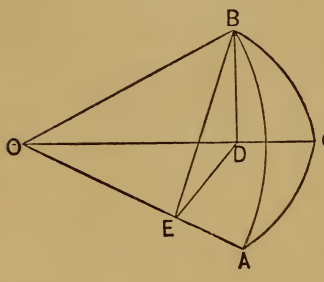
61. *A spherical triangle* is a three-sided figure on the surface of a sphere, each side being the arc of a circle. But since the sides of spherical triangles in ordinary practice are arcs of great circles, we will consider that —*A spherical triangle is formed by the arcs of three great circles.*

A great circle is one whose plane embraces the centre of the sphere. All other circles are called *small circles.*

A great circle divides the surface of the sphere into two equal parts ; two great circles will generally divide the surface into four parts ; and three great circles into eight parts, or eight triangles.

The sides or angles of some of these eight triangles will exceed 180°, but in this treatise only those triangles will be considered in which each part is less than 180°.

The centre O, of the sphere being common to the planes of all the great circles, the intersections of these planes necessarily radiate from that point, forming a trihedral $O-ACB$, having the three facial angles AOC, COB, AOB, and three dihedral angles, one at each of the edges of the trihedral. If with O as a centre and a common radius OA, arcs AB, AC, BC, be described, they will be on the surface of a sphere, and measure the facial angles which they subtend.



The dihedral angle between the planes AOC and AOB will be the same as the angle at A between the arcs AB and AC; for each will be measured by the angle between two lines, one in each plane, respectively perpendicular to the common intersection OA, drawn at A or at any other point of OA.

Spherical trigonometry, which is the science of the spherical triangle, may also be defined as the science of the trihedral.

It is proved in geometry, that in the spherical triangle which we are to consider :

The sum of the three angles may be anything between 180° and 540°.

The sum of the three sides is between 0° and 360°.

Each side is less than the sum of the other two.

Each side is the supplement of the angle opposite in the polar triangle.

Each angle is the supplement of the side opposite in the polar triangle.

The pole of an arc is the point where the axis of its great circle pierces the spherical surface. The arc of a great circle may be described by revolving one end of a rigid quadrant of a great circle about any point as a pole ; the remote end will trace the required arc. A string may be used in place of the rigid arc.

EXERCISES.

1. Describe the arc of a great circle on a sphere.

   [If no sphere be at hand, one may be extemporized by using an apple, ball, or some object only approximately spherical, and the solutions indicated.]

2. Describe two arcs of great circles mutually perpendicular.

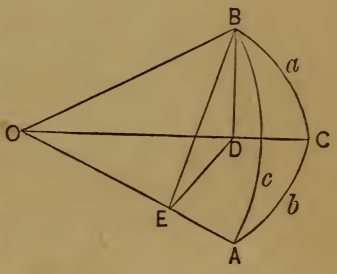
[The pole of the second arc will be at some point on the first arc.]

3. Construct three mutually perpendicular arcs of great circles.
[They will form a tri-rectangular triangle.]
4. Construct two mutually perpendicular arcs of great circles, and cut them by a third great circle perpendicular to one and oblique to the other.
5. Construct two mutually perpendicular arcs of great circles, and cut them by a third oblique to both the former.
6. Construct a spherical triangle in which two sides are equal and each of the sides oblique to the other two.
7. Pass a great circle through three given points.
8. Given a spherical triangle; describe its polar.
9. Given a spherical triangle ; draw a perpendicular from one angle to the side opposite.
10. Describe a spherical triangle each of whose sides and angles exceeds 90°.
11. Given $a = 43° 25'$, $c = 68° 13'$, $B = 135° 47'$, to find its polar.
12. Given $A = 115° 15'$, $B = 125° 25'$, $C = 137° 37'$, to find its polar.
13. Find the polar of the triangle $A = 1° 5'$, $a = 3° 7'$, $b = 175° 2'$.
14. Find the polar of the triangle $A = 188°$, $a = 187°$, $b = 5°$.

CHAPTER IV.

OF RIGHT ANGLED SPHERICAL TRIANGLES.

62. Let ABC be a spherical triangle in which C is a right angle. The side c opposite the right angle is called the hypothenuse, the other two sides a and b, the legs of the triangle, and A and B, the oblique angles. Let O be the centre of the sphere, then will OA, OB, OC, be radii, whose lengths are each equal to unity. From B drop the perpendicular BD to the radius OC, and from D a perpendicular to the radius OA, and join B and E; then will BE also be perpendicular to OA (Geometry). Since EB and ED are respectively in the planes AOB and AOC, and perpendicular to their line of intersection, the angle between them is the angle of the dihedral, and therefore equal to A, an angle of the triangle. According to article 30, we have



$$BD = \sin a,\ OD = \cos a,\ BE = \sin c,\ OE = \cos c;$$

and according to article 5, and the values just written,

$$ED = EB \cos A = \sin c \cos A \qquad (a)$$
$$ED = BD \cot A = \sin a \cot A \qquad (b)$$
$$ED = OE \tan b = \cos c \tan b \qquad (c)$$
$$ED = OD \sin b = \cos a \sin b \qquad (d)$$

Placing equations (a) and (b) equal, and reducing by means of equation (2), we have

$$\sin a = \sin c \sin A \qquad (123)$$

similarly, $$\sin b = \sin c \sin B \qquad (124)$$

Equation (123) is written from (124) by substituting b for a and B for A. This may be done, for in the former equation a is one leg of the triangle opposite the oblique angle A; but as it is immaterial which leg and angle opposite are used, we may substitute one set for the other.

From equations (a) and (c),

$$\cos A = \tan b \cot c \qquad (125)$$

similarly, $$\cos B = \tan a \cot c \qquad (126)$$

From (a) and (d),

$$\sin c \cos A = \cos a \sin b \qquad (e)$$

From (b) and (d),

$$\sin b = \tan a \cot A \qquad (127)$$

similarly, $$\sin a = \tan b \cot B \qquad (128)$$

From (c) and (d),

$$\cos c = \cos a \cos b \qquad (129)$$

Substituting (127) in (e) gives

$$\cos A = \cos a \sin B \qquad (130)$$

similarly, $$\cos B = \cos b \sin A \qquad (131)$$

Multiplying these together and comparing with (129) gives

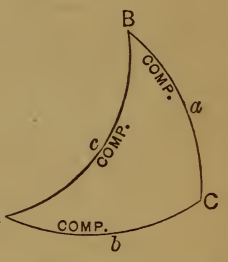
$$\cos c = \cot A \cot B. \qquad (132)$$

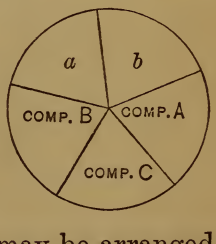
---

63. These formulas may be associated with the corresponding ones of plane triangles, and thus be more easily remembered. If the arcs be very short, we have approximately, $\sin a = a$, etc., $\tan a = a$, etc., $\cos a = 1$, nearly, etc.

In plane right triangles.		*In spherical right triangles.*	
$\sin A = \frac{a}{c}$	$\sin B = \frac{b}{c}$	$\sin A = \frac{\sin a}{\sin c}$	$\sin B = \frac{\sin b}{\sin c}$
$\cos A = \frac{b}{c}$	$\cos B = \frac{a}{c}$	$\cos A = \frac{\tan b}{\tan c}$	$\cos B = \frac{\tan a}{\tan c}$
$\tan A = \frac{a}{b}$	$\tan B = \frac{b}{a}$	$\tan A = \frac{\tan a}{\sin b}$	$\tan B = \frac{\tan b}{\sin a}$
$\sin A = \cos B$	$\sin B = \cos A$	$\sin A = \frac{\cos B}{\cos b}$	$\sin B = \frac{\cos A}{\cos a}$
$c^2 = a^2 + b^2$		$\cos c = \cos a \cos b$	
$1 = \cot A \cot B$		$\cos c = \cot A \cot B$	

64. **Napier's circular parts.** The above equations are all that are necessary for the complete solution of right triangles. Baron Napier, an eminent mathematician, made them so symmetrical that they may all be stated by two simple rules. Taking the complements of the two oblique angles, $90° - A = A'$, and $90° - B = B'$, and of the hypothenuse, $90° - c = c'$, the five quantities a, b, c', A', B', are called *Napier's circular parts.* It will be found by examining any right triangle, after neglecting the right angle that if any three parts be taken at random, one will either be between two adjacent parts or between two parts as remote as possible from it. The two former are called the *adjacent extremes*, the two latter the *opposite extremes*, and the third the *middle part.* The parts may be arranged within a circle as in the annexed figure, by means of





which the middle part and the adjacent extremes, or opposite extremes, as the case may be, may be readily selected.

EXERCISES.

1. Name Napier's *circular parts*.
2. If a, b, comp. A, be given, which will be the middle part? and will the extremes be adjacent or opposite?
3. If c' be the middle part, which will be the adjacent extremes, and which the opposite extremes?
4. If b, B, c, be given, which will be the middle part, and which the extremes? Will the extremes be adjacent or opposite?
5. If a, B, A, are the parts, which is the middle part? and which the extremes? Are they adjacent or opposite?

Napier's Rules.

65. Substituting for A, B, c, respectively $90° - A'$, $90° - B'$, $90° - c'$, and reducing, the preceding equations may be written and arranged as follows:

$$\left.\begin{array}{l} \sin\ a = \cos c' \cos A' = \tan b\ \tan B' \\ \sin\ b = \cos c' \cos B' = \tan a \tan A' \\ \sin c' = \cos a\ \cos b = \tan A' \tan B' \\ \sin A' = \cos a \cos B' = \tan b \tan c' \\ \sin B' = \cos b \cos A' = \tan a \tan c' \end{array}\right\} (C)$$

These stated as follows are *Napier's Rules:*

The sine of the middle part equals the product of the cosines of the opposite extremes.

The sine of the middle part equals the product of the tangents of the adjacent extremes.

To aid the memory in recalling these rules, it may be observed that the vowels are the same in the words *sine* and *mid*, and *cos* and *op*, also in the words *tan* and *adj.*

EXERCISES.

1. If $a = 120°$, $b = 100°$, and B is to be found, which equation of group (C) should be used, and what are the circular parts?
2. If $A = 170°$, $B = 175°$, and c is to be found, which are the circular parts and which formula should be used?
3. If $c = 90°$, find A and B, also a and b.
4. If $A = 45°$, and $B = 45°$, find c.
5. If $b = 45°$, and $A = 45°$, find B.
6. If $B = 135°$, and $b = 45°$, find a.
7. If $B = 90°$, and $a = 20°$, find A and b.
8. If the five parts are—the oblique angles, the hypothenuse, and the complements of the sides; deduce two rules similar to Napier's.

   [They are called Maudit's Rules.]

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66. *The species of the parts* are their relations to 90°. If two parts are both less than 90° they are of the same species; also if both are between 90° and 180°. But if one is between 0° and 90°, and the other between 90° and 180°, they are of different species. When the result is found in terms of the sine of the angle, it will be ambiguous unless the particular angle can be determined by its relation to the given parts.

In a right angled spherical triangle, if the hypothenuse is less than 90°, *the two sides and the two oblique angles are, respectively, of the same species.*

For, from the third of group (C) we will have $c' < 90°$, and sin c' *positive;* hence

$$\cos a \cos b$$

must be positive, and therefore cos a and cos b must have the same sign; hence if a is less than 90° b must also be less. Similarly, from the same equation,

$$\tan A' \tan B', \text{ or } \cot A \cot B,$$

must be positive and A and B must both be less or both greater than 90°.

In a right spherical triangle, if the hypothenuse exceeds 90° *the two sides and the two oblique angles will, respectively, be of different species.*

For, in the third of group (C), $\sin c' = \cos c$ will be negative, hence $\cos a$ and $\cos b$ will have opposite signs; and similarly in regard to A and B.

In a right spherical triangle, an angle and its opposite side are of the same species.

For, by the fifth of group (C), we have

$$\sin A = \frac{\cos B}{\cos b}, \tag{133}$$

and since $\sin A$ will be positive ($A < 180°$), $\cos B$ and $\cos b$ must have the same sign, and hence the angles will be both less or both greater than 90°.

The solution of right spherical triangles may be classed under six cases, as follows:

67. **Case I.** *Given the hypothenuse and a side,* as c and a.

To find A. Here a is the middle part, and from the first of group (C) we have

$$\sin A = \frac{\sin a}{\sin c}$$

or, $$\log \sin A = \log \sin a - \log \sin c \tag{134}$$

To find B. In this case B is the middle part and the fifth of group (C) gives

$$\cos B = \tan a \cot c$$

or, $$\log \cos B = \log \tan a + \log \cot c \tag{135}$$

To find b. Here c is the middle part, and the third of (C) gives

$$\cos b = \frac{\cos c}{\cos a}$$

or,
$$\log \cos b = \log \cos c - \log \cos a \qquad (136)$$

Check. The third of (C) gives

$$\cos B = \cos b \sin A$$

or,
$$\log \cos B = \log \cos b + \log \sin A \qquad (137)$$

EXAMPLES.

1. Given $a = 20°$ $c = 140°$.

	By (134) log	By (135) log	By (136) log
$a = 20°$	sin 9.534052	tan 9.561066	ar co cos 0.027014
$c = 140°$	ar co sin 0.191932	cot 10.076187n	cos 9.884254n
	A, sin 9.725984	B, cos 9.637253n	b, cos 9.911268n

Check, By (137), A, sin 9.725984

B, cos 9.637252n

which being the same as before found checks the work. Taking the angles from a table of logarithmic functions gives

Ans. $A = 32° 8' 48''$, $B = 115° 42' 23''$, $b = 144° 36' 28''$.

[n implies that the function is negative; thus cot 140° is negative. Since the sign of the result depends upon the number of negative factors, if there be but one n the result will be negative, and, similarly, if there be an odd number of n's in the logarithms added, the result will be negative; but if there be an even number of n's the sign of the result will be positive.

The last figure or figures of the logarithm of the "check" may not agree exactly with that previously found, and yet the work may be correct, for the last figure of a logarithm in the table is not always correct.]

2. Given $a = 141° 11'$, $c = 127° 12'$.
Ans. $A = 128° 5' 54''$, $B = 52° 21' 45''$, $b = 39° 6' 23''$.

3. Given $b = 18° 01' 50''$, $c = 86° 51'$.
Ans. $A = 88° 58' 25'$, $B = 18° 03' 32''$, $a = 86° 41' 14''$.

---

68. **Case II.** *Given one angle and its opposite side,* as A and a.

To find c. By Napier's Rules,

$$\sin c = \frac{\sin a}{\sin A} \tag{138}$$

To find b,

$$\sin b = \tan a \cot A \tag{139}$$

To find B,

$$\sin B = \frac{\cos A}{\cos a} \tag{140}$$

Check,

$$\sin b = \sin c \sin B \tag{141}$$

In this case the ambiguity is not removed by the tests in article 66, and hence there will, generally, be two solutions.

EXAMPLES.

1. Given $B = 150°$, $b = 160°$.

Ans. $\left.\begin{array}{l} c = 136° 50' 23'' \\ a = 39° 4' 50'' \\ A = 67° 9' 42'' \end{array}\right\}$ or $\left\{\begin{array}{l} c = 43° 9' 36'' \\ a = 140° 55' 9'' \\ A = 112° 50' 17'' \end{array}\right.$

According to article 66, a and A must be in the same quadrant.

2. Given $A = 37° 28'$, $a = 35° 44'$.

Ans. $c = 73° 45' 15''$, $B = 77° 54'$, $b = 69° 50' 24''$ or $c = 106° 14' 45''$, $B = 102° 6'$, $b = 110° 9' 36''$.

3. Given $A = 104° 59'$ $a = 129° 33'$.

Ans. $c = 127° 2' 27''$, $B = 23° 57' 19''$, $b = 18° 54' 38''$ or $c = 52° 57' 33''$, $B = 156° 2' 41''$, $b = 161° 5' 22''$.

4. Given $B = 80° 1'$, $b = 67° 36'$.

5. Given $B = 45°$, $b = 45°$.

---

69. **Case III.** *Given the hypothenuse and one angle.*

[The student may select the formulas for this and the remaining cases from group (C).]

EXAMPLES.

1. Given $A = 23° 28'$, $c = 145°$.

Ans. $B = 109° 34' 33''$, $a = 13° 12' 12''$, $b = 147° 17' 15''$.

2. Given $B = 50° 8' 21''$, $c = 32° 34'$.

Ans. $A = 44° 44'$, $a = 22° 15' 43''$, $b = 24° 24' 19''$.

3. Given $B = 80° 55' 27''$, $c = 98° 6' 43''$.

Ans. $A = 131° 27' 18''$, $a = 132° 6'$, $b = 77° 51'$.

---

70. **Case IV.** *Given an angle and a side adjacent.*

EXAMPLES.

1. Given $a = 118° 54'$, $B = 12° 19'$.

Ans. $c = 118° 20' 20''$, $A = 95° 55' 2''$, $b = 10° 49' 17''$.

2. Given $b = 54° 30'$, $A = 35° 30'$.
Ans. $c = 59° 51' 20''.8$, $a = 30° 8' 39''.2$, $B = 70° 17' 35''$.

3. Given $B = 137° 24' 21''$, $a = 29° 46' 08''$.
Ans. $b = 155° 27' 54''$, $c = 142° 9' 13''$, $A = 54° 1' 16''$.

---

71. Case V. *Given the two sides, a and b.*

EXAMPLES.

1. Given $a = 56° 34'$, $b = 27° 18'$.
Ans. $A = 16° 50' 47''$, $B = 31° 44' 9''$, $c = 60° 41' 9''$.

2. Given $a = 144° 27' 03''$, $b = 32° 8' 56''$.
Ans. $A = 126° 40' 24''$, $B = 47° 13' 43''$, $c = 133° 32' 26''$

3. Given $a = 32° 9' 17''$, $b = 32° 41'$.
Ans. $A = 49° 20' 17''$, $B = 50° 19' 16''$, $c = 44° 33' 17''$.

---

72. Case VI. *Given the two oblique angles, A and B.*

EXAMPLES.

1. Given $A = 91° 11'$, $B = 111° 11'$.
Ans. $c = 89° 32' 29''$, $a = 91° 16' 8''$, $b = 111° 11' 16''$

2. Given $A = 67° 54' 47''$, $B = 99° 57' 35''$.
Ans. $a = 67° 33' 27''$, $b = 100° 45'$, $c = 94° 5'$.

3. Given $A = 54° 01' 15''$, $B = 137° 24' 21''$.
Ans. $a = 29° 46' 08''$, $b = 155° 27' 55''$, $c = 142° 09' 12''$.

---

73. *A quadrantal triangle* is one having one or more of its sides equal to a quadrant. Its polar triangle will be right angled; hence to solve a quadrantal triangle pass to its polar, solve that, and pass back to the original triangle.

EXAMPLES.

1. Given $c = 90°$, $B = 74° 45'$, $a = 18° 12'$, to find the remaining parts.
   Ans. $C = 104° 31' 13''$, $b = 4° 42' 15''$, $A = 71° 10' 54''$.

2. Given $c = 90°$, $A = 42° 01'$, $B = 121° 20'$ to find the remaining parts.
   Ans. $C = 66° 57' 15''$, $b = 111° 50' 18''$, $a = 45° 40' 17''$.

---

74. If the angle sought be near zero, or ninety degrees, it may sometimes be found more accurately with ordinary logarithmic tables by means of special formulæ, a few of which are here given without proof.

$$\tan^2 (45° - \tfrac{1}{2}A) = \tan \tfrac{1}{2}(c - a) \cot \tfrac{1}{2}(c + a)$$
$$\tan^2 \tfrac{1}{2}b = \tan \tfrac{1}{2}(c - a) \tan \tfrac{1}{2}(c + a)$$
$$\tan^2 \tfrac{1}{2}B = \sin (c - a) \csc \tfrac{1}{2}(c + a) \qquad [56, 131, 30 \div 31]$$
$$\tan^2 \tfrac{1}{2}c = -\cos (A + B) \sec (A - B) \qquad [56, 129, 32 \div 33]$$
$$\tan^2 (45° - \tfrac{1}{2}c) = \tan \tfrac{1}{2}(A - a) \cot \tfrac{1}{2}(A + a)$$

To obtain the first of these, we have from equation (123)

$$\sin A = \frac{\sin a}{\sin c};$$

$$\therefore \frac{1 - \sin A}{1 + \sin A} = \frac{\sin c - \sin a}{\sin c + \sin a} = \tan \tfrac{1}{2}(c - a) \cot \tfrac{1}{2}(c + a).$$

In equation (66) make $x = 90°$, and it will readily be found that the left member of the above equation reduces to $\tan^2 (45° - \frac{1}{2}A)$.

The fifth equation above may be found in a similar manner by beginning with equation (123) after writing it thus :

$$\sin c = \frac{\sin a}{\sin A}.$$

To obtain the second equation above:

[Eq. (56)] $$\tan^2 \tfrac{1}{2}b = \frac{1 - 2 \cos b + \cos^2 b}{1 - \cos^2 b},$$

[Eq. (129)] $$\cos^2 \tfrac{1}{2}b = \cos^2 c \div \cos^2 a,$$

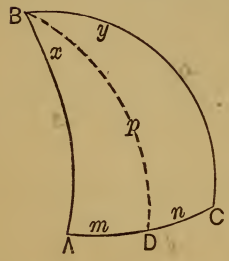
which substituted in the preceding and reduced by the aid of (67) multiplied by (69) will at once give the required result.

CHAPTER V.

OBLIQUE ANGLED TRIANGLES SOLVED BY MEANS OF RIGHT ANGLED TRIANGLES.

75. *In a spherical triangle the sines of the angles are proportional to the sines of the sides opposite.*

In the spherical triangle ABC, let fall the perpendicular BD from any angle B upon the opposite side, AC, thus forming two right angled triangles ADB and CDB; of which let m be the base of one, and n that of the other, x the angle opposite m and y that opposite n, as in the figure. According to Napier's rules, we have for the



triangle ADB, $$\sin p = \sin c \sin A \qquad (142)$$

triangle CDB, $$\sin p = \sin a \sin C \qquad (143)$$

Therefore,

$$\left.\begin{aligned} &\sin a \sin C = \sin c \sin A \\ \text{similarly,}\quad &\sin b \sin A = \sin a \sin B \\ &\sin c \sin B = \sin b \sin C \end{aligned}\right\} \qquad (144)$$

These may be written

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \qquad (145)$$

which was to be proved.

76. *To find the segments m and n, given the three sides.*
From Napier's first rule,

$$\cos a = \cos p \cos n$$
$$\cos c = \cos p \cos m$$

Divide the former by the latter; then after adding each term to unity, and subtracting each term from unity, divide the latter by the former, and find

$$\frac{\cos c - \cos a}{\cos c + \cos a} = \frac{\cos m - \cos n}{\cos m + \cos n}$$

But, (69),

$$\frac{\cos c - \cos a}{\cos c + \cos a} = \tan \tfrac{1}{2}(a + c) \tan \tfrac{1}{2}(a - c)$$

and

$$\frac{\cos m - \cos n}{\cos m + \cos n} = \tan \tfrac{1}{2}(n + m) \tan \tfrac{1}{2}(n - m)$$

$$\therefore \tan \tfrac{1}{2}(a + c) \tan \tfrac{1}{2}(a - c) = \tan \tfrac{1}{2}b \tan \tfrac{1}{2}(n - m) \quad (146)$$

from which $n - m$ may be found; and since $n + m = b$, the values of n and m become known.

By means of Napier's rules and equation (146) all the cases of oblique triangles may be solved, and the solution may sometimes be facilitated by means of equations (144).

The perpendicular BD may fall entirely without the triangle, and if it falls to the right of BC, n will be entirely on the prolongation of the base, and m will be the entire base *plus* the elongation. This perpendicular will also meet the great circle of the base in two points, distant from each other 180°, and if that perpendicular be taken whose foot is nearest to one end or the other of the base, then will $m + n$ be numerically less than 180°, as it should. If n be the external segment, it will be considered as *minus*, so that, algebraically, we will have in all cases $m + n = b$, which, numerically, may sometimes be $m - n = b$.

Since the greater segment is not necessarily adjacent to the greater

side, its position must be determined. This may be done by means of equation (146); for the signs of all the factors except the last will be determined from the data, and therefore the sign of $\tan \frac{1}{2}(n - m)$ becomes known, and since $\frac{1}{2}(n - m)$ will be numerically less than 90°, it follows that when $\tan \frac{1}{2}(n - m)$ is +, $n > m$, and if it be −, $n < m$.

In the solution of oblique spherical triangles, we may have six cases, as follows:

77. **Case I.** *Given two sides and the included angle,* as a, b, C.

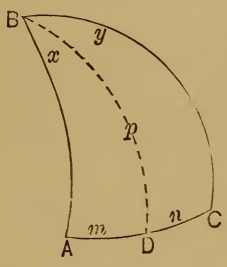
From the angle B drop the perpendicular p on the side opposite; then find p by means of equation (143).

Find the segment n by means of the second of Napier's rules (observing the species), which will give

$$\tan n = \frac{\cos C}{\tan a}; \tag{147}$$

then

$$m = b - n \tag{148}$$



Knowing m and p, find A by Napier's first rule, and c by the second, giving

$$\cot A = \frac{\sin m}{\tan p}, \tag{149}$$

$$\cos c = \cos p \cos m \tag{150}$$

Find B by means of $(144)_2$; or find x and y by means of the right triangles, in which case $B = x + y$.

---

78. **Case II.** *Given two angles and the included side.* Pass to the polar and solve by Case I., then pass

back. Or, given C, B, a, by means of Napier's rules, find n, p, y; then $x = B - y$, and with x and p complete the solution.

---

79. **Case III.** *Given two sides and an angle opposite one of them*, as a, c, A.

Find the angle C by means of $(144)_1$. Then by Napier's rules

$$\cos A = \tan c \tan m \qquad (151)$$

$$\cos C = \tan a \cot n \qquad (152)$$

by means of which m and n may be found, then

$$b = m + n,$$

after which B may be found by means of equation $(144)_2$.

This case admits of two solutions, one or none, which may be shown in the same manner as in article 52, Case II., of plane triangles.

If $a > p$, and $a < c$, there will be two triangles.

If $a = p$, there will be one triangle.

If $a < p$, there will be no triangle.

If $a = c$, there will be one triangle.

If $a > c$, and $A < 90°$ or $A > 90°$, there will be one triangle.

If $a > c$ and $A = 90°$, there will be two triangles.

---

80. **Case IV.** *Given two angles and a side opposite one of them*, as A, C, a.

Pass to the polar, solve that by Case III., and then pass back. Or, solve directly, finding c by $(144)_1$, m and n by (151) and (152); then $b = m + n$, after which B may be found by $(144)_2$.

81. **Case V.** *Given the three sides.*

By means of (148) we have

$$\tan \tfrac{1}{2}(n - m) = \tan \tfrac{1}{2}(a + c) \tan \tfrac{1}{2}(a - c) \cot \tfrac{1}{2}(n + m), \quad (153)$$

from which find n and m; after which the solution may be computed by Napier's rules.

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82. **Case VI.** *Given the three angles.*

Pass to the polar, solve that by Case V., and then pass back.

EXAMPLES.

1. Given $A = 92°\ 10'$, $B = 72°\ 15'$, and $C = 135°\ 15'\ 20''$, to solve the triangle.

This comes under case VI., and we have, passing to the polar,

$$a' = 87°\ 50',\ b' = 107°\ 45',\ c' = 44°\ 47'\ 40''.$$

		By (153)	
$\frac{1}{2}(a' + c') =$	$66°\ 17'\ 20''$	log tan	10.357337
$\frac{1}{2}(a' - c') =$	$21°\ 32'\ 40''$	log tan	9.596384
$\frac{1}{2}(n + m) =$	$53°\ 52'\ 30''$	log cot	9.863252
$\frac{1}{2}(n - m) =$	$33°\ 16'\ 9''$	log tan	9.816973

		By (125)		By (125)	
$\therefore n =$	$87°\ 08'\ 39''$	log tan	11.302022		
$m =$	$20°\ 36'\ 21''$			log tan	9.575178
$a' =$	$87°\ 50'\ 00''$	log cot	8.577877		
$c' =$	$44°\ 44'\ 40''$			log cot	10.003875
$C' =$	$40°\ 40'\ 35''$	log cos	9.879899		
$A' =$	$67°\ 42'\ 21''$			log cos	9.579053

To find B'.

		By (132)	By (132)
$a' =$	$87° 50' 00''$	log sec 11.422434	
$C' =$	$40° 40' 35''$	log cot 10.065795	
$c' =$	$44° 44' 40''$		log sec 10.148587
$A' =$	$67° 42' 2''$		log cot 9.612795
$y =$	$88° 08' 21''$	log tan 11.488229	
$x =$	$29° 59' 48''$		log tan 9.761382
$C' =$	$118° 08' 09''$		

Passing back we find

Ans. $\begin{cases} a = 112° 17' 39'' \\ b = 139° 19' 25'' \\ c = 61° 51' 51'' \end{cases}$

2. Given $a = 62°$, $b = 75°$, $C = 100°$.
3. Given $A = 150°$, $b = 60°$ $C = 120°$.
4. Given $A = 50° 12'$, $B = 58° 08'$, $a = 62° 42'$, to find the other parts.

Ans. $\begin{cases} b = 79° 12' 10'', & c = 119° 03' 26'', & C = 130° 54' 28'' \\ \text{or } b = 100° 47' 50'', & c = 152° 14' 18'', & C = 156° 15' 6'' \end{cases}$

5. Given $a = 90°$, $b = 90°$, $c = 90°$.
6. Given $a = 45°$, $b = 45°$, $c = 45°$.
7. Given $a = 175°$, $b = 175°$, $c = 5°$.
8. Given $a = 84° 14' 29''$, $b = 44° 13' 45''$, and $A = 130° 5' 22''$ to find the other parts.

Ans. $B = 32° 26' 6\frac{2}{3}''$, $C = 36° 45' 28''$, $c = 51° 6' 12''$.

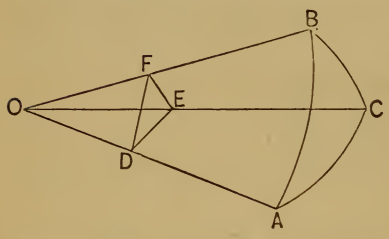
9. Given the two sides $44° 13' 45''$ and $84° 14' 29''$, and the included angle $36° 45' 28''$; to find the other parts.

Ans. Angles $32° 26' 6''$, and $130° 5' 22''$, side $51° 6' 12''$.

CHAPTER VI.

GENERAL FORMULÆ.

83. *Value of the cosine of a side of a spherical triangle.* Let ABC be a spherical triangle, O the centre of the sphere, $O - ABC$ the trihedral. Conceive the trihedral to be cut by a plane perpendicular to the edge OA, passing through any point D, and let DF, DE, EF, be the intersections with its faces. Then will DF and DE be perpendicular to OA, the triangles ODF and ODE will be right, and the angle $EDF = A$.



We have, article 55,

$$EF^2 = OE^2 + OF^2 - 2OE \,.\, OF \cos a \qquad (a)$$

$$EF^2 = DE^2 + DF^2 - 2DE \,.\, DF \cos A \qquad (b)$$

$$OD^2 = OE^2 - DE^2 \qquad (c)$$

$$OD^2 = OF^2 - DF^2 \qquad (d)$$

Subtracting (b) from (a) and reducing by (c) and (d), gives

$$\cos a = \frac{OD}{OE} \cdot \frac{OD}{OF} + \frac{DE}{OE} \cdot \frac{DF}{OF} \cos A.$$

Substituting the trigonometrical functions, group (A),

$$\left.\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C \end{aligned}\right\} \quad (154)$$

To aid in memorizing these, observe that the first and last are *cos* of letter of same name, thus cos a and cos A, the former small the latter large; also that cos A is joined to the factors of sines of the other sides.

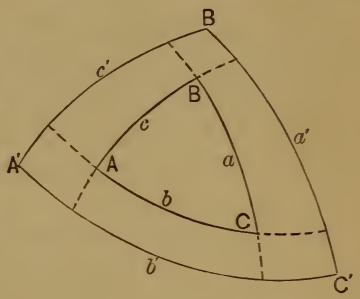
Also observe that the second equation may be written from the first by advancing the letters one in the scale a, b, c, a; thus, for a write b, for b, c, and for c write a. Similarly the third may be found from the second.

84. Let $A'B'C'$ be a spherical triangle polar to ABC, then by geometry,

$A' = 180° - a$ $\quad a' = 180° - A$

$B' = 180° - b$ $\quad b' = 180° - B$

$C' = 180° - c$ $\quad c' = 180° - C$



The first of (154) applied to the triangle $A'B'C'$, gives

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A',$$

in which substituting the above values, we have

$$-\cos A = (-\cos B)(-\cos C) + \sin B \sin C (-\cos a)$$

or

$$\left.\begin{aligned} \cos A &= -\cos B \cos C + \sin B \sin C \cos a \\ \cos B &= -\cos C \cos A + \sin C \sin A \cos b \\ \cos C &= -\cos A \cos B + \sin A \sin B \cos c \end{aligned}\right\} \quad (155)$$

Thus, by the use of the polar triangle, formulæ may be obtained in which the functions of the angles and sides may be found in place of the sides and angles.

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85. Substituting $\cos a$ from $(155)_1$, in $(154)_2$, we have

$$\cos b = \cos^2 c \cos b + \sin b \cos c \sin c \cos A + \sin a \sin c \cos B$$

But,

$$\cos^2 c \cos b = (1 - \sin^2 c) \cos b = \cos b - \cos b \sin^2 c$$

which substituted and dividing by $\sin c$, gives

$$\left.\begin{aligned} \sin a \cos B &= \sin c \cos b - \sin b \cos c \cos A \\ \sin b \cos C &= \sin a \cos c - \sin c \cos a \cos B \\ \sin c \cos A &= \sin b \cos a - \sin a \cos b \cos C \end{aligned}\right\} \quad (156)$$

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86. From $(144)_2$ we have

$$\frac{\sin a \sin B}{\sin A} = \sin b$$

which, divided into $(156)_1$ member by member, we have

$$\left.\begin{aligned} \sin A \cot B &= \sin c \cot b - \cos c \cos A \\ \text{and,} \quad \sin B \cot C &= \sin a \cot c - \cos a \cos B \\ \sin C \cot A &= \sin b \cot a - \cos b \cos C \end{aligned}\right\} \quad (157)$$

In the triangle ABC, A and B may be interchanged provided that a and b are also interchanged. Interchanging these letters in the precedi g equations, gives,

$$\left.\begin{aligned} \sin B \cot A &= \sin c \cot a - \cos c \cos B \\ \sin A \cot C &= \sin b \cot c - \cos b \cos A \\ \sin C \cot B &= \sin a \cot b - \cos a \cos C \end{aligned}\right\} \quad (158)$$

In these equations the permutations may be made by passing backward with the letters, thus, C, B, A, C, or by beginning with the third equation and writing the 2d from it by permuting the letters in their natural order, and the first from the second in the same manner.

---

87. To transform (154) and (155) so as to adapt them to logarithmic computation.

In (154) substitute

Eq. (46), $\cos A = 1 - 2 \sin^2 \frac{1}{2}A$

then

$$\cos a = \cos (b - c) - 2 \sin b \cos c \sin^2 \tfrac{1}{2}A$$

$$\therefore 2 \sin^2 \tfrac{1}{2}A = \frac{\cos (b - c) - \cos a}{\sin b \sin c} \quad (159)$$

If $x = a$, and $y = b - c$

then

$\frac{1}{2}(x + y) = \frac{1}{2}(a + b - c)$, $\frac{1}{2}(x - y) = \frac{1}{2}(a - b + c)$

and (Eq. (65)),

$\cos (b - c) - \cos a = 2 \sin^2 \frac{1}{2}(a - b + c) \sin \frac{1}{2}(a - b + c)$,

which substituted in (159) gives

$$\sin^2 \tfrac{1}{2}A = \frac{\sin \frac{1}{2}(a - b + c) \sin \frac{1}{2}(a + b - c)}{\sin b \sin c} \quad (160)$$

Let s be one half the sum of the sides, that is,

$$a + b + c = 2s$$

then $a - b + c = a + b - 2b + c = 2(s - b)$

$a + b - c = a + b + c - 2c = 2(s - c)$

which substituted in (160) gives

$$\left.\begin{aligned} \sin^2 \tfrac{1}{2}A &= \frac{\sin(s-b)\ \sin(s-c)}{\sin b \sin c} \\ \text{similarly,}\quad \sin^2 \tfrac{1}{2}B &= \frac{\sin(s-c)\ \sin(s-a)}{\sin c \sin a} \\ \sin^2 \tfrac{1}{2}C &= \frac{\sin(s-a)\ \sin(s-b)}{\sin a \sin b} \end{aligned}\right\} \quad (161)$$

Passing to the polar triangle,

$$\left.\begin{aligned} \cos^2 \tfrac{1}{2}a &= \frac{\sin(S-B)\ \sin(S-C)}{\sin B \sin C} \\ \cos^2 \tfrac{1}{2}b &= \frac{\sin(S-C)\ \sin S-A)}{\sin C \sin A} \\ \cos^2 \tfrac{1}{2}c &= \frac{\sin(S-A)\ \sin(S-B)}{\sin A \sin B} \end{aligned}\right\} \quad (162)$$

Again, substituting,

Eq. (47), $\cos A = 2\cos^2 \tfrac{1}{2}A - 1$

in $(154)_1$ gives

$$\cos a = \cos(b + c) + 2 \sin b \sin c \cos^2 \tfrac{1}{2}A\ ;$$

$$\therefore\ \cos^2 \tfrac{1}{2}A = \frac{\cos a - \cos(b + c)}{2 \sin b \sin c}$$

or

$$\left.\begin{aligned} \cos^2 \tfrac{1}{2}A &= \frac{\sin s \sin(s-a)}{\sin b \sin c} \\ \text{similarly,}\quad \cos^2 \tfrac{1}{2}B &= \frac{\sin s \sin(s-b)}{\sin c \sin a} \\ \cos^2 \tfrac{1}{2}C &= \frac{\sin s \sin(s-c)}{\sin a \sin b} \end{aligned}\right\} \quad (163)$$

Passing to the polar triangle,

$$\left.\begin{aligned} \sin^2 \tfrac{1}{2}a &= \frac{-\cos S \cos (S-A)}{\sin B \sin C} \\ \sin^2 \tfrac{1}{2}b &= \frac{-\cos S \cos (S-B)}{\cos C \cos A} \\ \sin^2 \tfrac{1}{2}c &= \frac{-\cos S \cos (S-C)}{\cos A \cos B} \end{aligned}\right\} \quad (164)$$

---

88. From (161) and (163) we find, observing that $\sin \frac{1}{2}A \div \cos \frac{1}{2}A = \tan \frac{1}{2}A$,

$$\left.\begin{aligned} \tan^2 \tfrac{1}{2}A &= \frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)} \\ \tan^2 \tfrac{1}{2}B &= \frac{\sin (s-c) \sin (s-a)}{\sin s \sin (s-b)} \\ \tan^2 \tfrac{1}{2}C &= \frac{\sin (s-a) \sin (s-b)}{\sin s \sin (s-c)} \end{aligned}\right\} \quad (165)$$

Passing to the polar triangle; or by means of (164) and (162) we have

$$\left.\begin{aligned} \tan^2 \tfrac{1}{2}a &= \frac{-\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)} \\ \tan^2 \tfrac{1}{2}b &= \frac{-\cos S \cos (S-B)}{\cos (S-C) \cos (S-A)} \\ \tan^2 \tfrac{1}{2}c &= \frac{-\cos S \cos (S-C)}{\cos (S-A) \cos (S-B)} \end{aligned}\right\} \quad (166)$$

---

89. Where several parts—as for instance all the angles—are to be found, it may be better to proceed as follows:

Let

$$k = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}} \tag{167}$$

then (165) become

$$\left.\begin{aligned} \tan \tfrac{1}{2}A &= \frac{k}{\sin(s-a)} \\ \tan \tfrac{1}{2}B &= \frac{k}{\sin(s-b)} \\ \tan \tfrac{1}{2}C &= \frac{k}{\sin(s-c)} \end{aligned}\right\} \tag{168}$$

Similarly, put

$$K = \sqrt{\frac{-\cos S}{\cos(S-A)\cos(S-B)\cos(S-C)}} \tag{169}$$

then (166) become

$$\left.\begin{aligned} \tan \tfrac{1}{2}a &= K\cos(S-A) \\ \tan \tfrac{1}{2}b &= K\cos(S-B) \\ \tan \tfrac{1}{2}c &= K\cos(S-C) \end{aligned}\right\} \tag{170}$$

---

90. **To deduce Napier's Analogies,** which are

$$\left.\begin{aligned} \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a-b)} &= \frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} \\ \frac{\cot \frac{1}{2}C}{\tan \frac{1}{2}(A-B)} &= \frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} \\ \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a+b)} &= \frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} \\ \frac{\cot \frac{1}{2}C}{\tan \frac{1}{2}(A+B)} &= \frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} \end{aligned}\right\} \tag{171}$$

Dividing $(165)_1$ by $(165)_2$ gives

$$\frac{\tan \frac{1}{2}A}{\tan \frac{1}{2}B} = \frac{\sin (s-b)}{\sin (s-a)}$$

By composition and division, we have

$$\frac{\tan \frac{1}{2}A + \tan \frac{1}{2}B}{\tan \frac{1}{2}A - \tan \frac{1}{2}B} = \frac{\sin (s-b) + \sin (s-a)}{\sin (s-b) - \sin (s-a)}$$

Equation (66),

$$= \frac{\tan \frac{1}{2}(s-b+s-a)}{\tan \frac{1}{2}(s-b-(s-a))}$$

$$= \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a-b)}$$

To reduce the left member, we have from (30) and (31)

$$\frac{\sin (x+y)}{\sin (x-y)} = \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y},$$

and dividing both numerator and denominator of the right member by $\cos x \cos y$ we find

$$\frac{\sin (x+y)}{\sin (x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$$

Making $x = \frac{1}{2}A$, and $y = \frac{1}{2}B$, we have

$$\frac{\tan \frac{1}{2}A + \tan \frac{1}{2}B}{\tan \frac{1}{2}A - \tan \frac{1}{2}B} = \frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a-b)}$$

which is $(171)_1$.

Passing to the polar triangle we have

$$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2}C}{\tan \frac{1}{2}(A-B)}$$

which is $(171)_2$.

Multiplying $(165)_1$ by $(165)_2$,

$$\tan \tfrac{1}{2}A \tan \tfrac{1}{2}B = \frac{\sin (s-c)}{\sin s}$$

or,

$$1 : \tan \tfrac{1}{2}A \tan \tfrac{1}{2}B :: \sin s : \sin (s-c),$$

which by composition and division,

$$\frac{1 - \tan \frac{1}{2}A \tan \frac{1}{2}B}{1 + \tan \frac{1}{2}A \tan \frac{1}{2}B} = \frac{\sin s - \sin (s-c)}{\sin s + \sin (s-c)}$$

Equation (66), $$= \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a+b)}$$

hence $$\frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a+b)}$$

which is $(171)_3$.

Passing to the polar gives

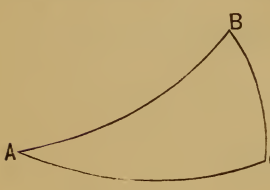
$$\frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = \frac{\cot C}{\tan \frac{1}{2}(A+B)}$$

which is $(171)_4$.

These equations are especially useful in solving a triangle in which two sides and the included angle, or two angles and included side, are given. In using them the species of the parts must be observed. In the first, $\tan \frac{1}{2}(a-b)$ and $\sin \frac{1}{2}(A-B)$ are necessarily of the same species, and therefore $\tan \frac{1}{2}c$ and $\sin \frac{1}{2}(A+B)$ must be of the same species. Similarly, in the second, $\tan \frac{1}{2}(A-B)$ and $\sin \frac{1}{2}(a-b)$ are of the same species, and therefore $\cot \frac{1}{2}C$ and $\sin \frac{1}{2}(a+b)$ must be of the same species.

By means of the general equations the solutions of the six cases may be made as follows:

91. Case I. *Given two sides and the included angle,* as a, b, C, to find the remaining parts.



To find the angles A and B.

From $(171)_2$ and $(171)_4$ we have

$$\tan \tfrac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \tfrac{1}{2}C \qquad (173)$$

$$\tan \tfrac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2}C \qquad (174)$$

by means of which $\frac{1}{2}(A - B)$ and $\frac{1}{2}(A + B)$ may be found, and of the last two expressions the sum is A, and the difference B.

To find c. From $(144)_1$ we have

$$\sin c = \frac{\sin C}{\sin A} \sin a \qquad (175)$$

EXAMPLE.

Given $a = 73° 58'$, $b = 38° 45'$, $C = 46° 33' 39''$.

Then *to find* A and B

		By (174)	By (175)
$\frac{1}{2}(a - b) =$	17° 36′ 30″	log cos 9.979160	log sin 9.480738
$\frac{1}{2}(a + b) =$	56° 21′ 30″	log sec 0.256493	log csc 0.079606
$\frac{1}{2} C =$	23° 16′ 49″.5	log cot 0.366266	log cot 0.366266
$\frac{1}{2}(A + B) =$	75° 57′ 33″.8	log tan 10.601919	
$\frac{1}{2}(A - B) =$	40° 10′ 54″.3		log tan 9.926610
$A =$	116° 8′ 28″.1		
$B =$	35° 46′ 39″.5		

To find c

		By (175)
$C =$	46° 33′ 39″	log sin 9.861060
$a =$	73° 58′	log sin 9 982769
$A =$	116° 8′ 28″.1	log csc 0.046863
$c =$	51° 00′ 15″.8	log sin 9.890532

Ans. $\begin{cases} A = 116° 8' 28''.1 \\ B = 35° 46' 39''.5 \\ c = 51° 00' 15''.8 \end{cases}$

---

92. Case II. *Given two angles and the included side, A, B, c.*

Pass to the polar, solve that by Case I., and then pass back.

Or, by means of $(171)_1$ and $(171)_3$ find a and b, thus:

$$\tan \tfrac{1}{2}(a-b) = \frac{\sin \tfrac{1}{2}(A-B)}{\sin \tfrac{1}{2}(A+B)} \tan \tfrac{1}{2}c \qquad (176)$$

$$\tan \tfrac{1}{2}(a+b) = \frac{\cos \tfrac{1}{2}(A-B)}{\cos \tfrac{1}{2}(A+B)} \tan \tfrac{1}{2}c \qquad (177)$$

Then find C by $(144)_2$,

$$\sin C = \frac{\sin c}{\sin a} \sin A. \qquad (178)$$

---

93. **Case III.** *Given two sides and the angle opposite one of them*, as a, b, A.

Find B from $(144)_2$, then c from $(171)_1$ and C from $(171)_2$. The equations are

$$\sin B = \sin A \sin b \csc a \qquad (179)$$

$$\tan \tfrac{1}{2}c = \frac{\sin \tfrac{1}{2}(A+B)}{\sin \tfrac{1}{2}(A-B)} \tan \tfrac{1}{2}(a-b) \qquad (180)$$

$$\cot \tfrac{1}{2}C = \frac{\sin \tfrac{1}{2}(a+b)}{\sin \tfrac{1}{2}(a-b)} \tan \tfrac{1}{2}(A-B) \qquad (181)$$

Equations (154) will give c and C directly by means of natural functions.

This case, like the corresponding one in plane triangles, may have two solutions, one solution, or no solution, as already shown in Article 79. There will be two solutions when

$$A < 90°, \qquad a + b < 180°, \qquad a < b,$$

or $$A > 90°, \qquad a + b > 180°, \qquad a > b.$$

If $\sin B = 1$ in (179), there will be one solution, and the side a will be perpendicular to the side b.

If $\sin B > 1$ there will be no solution.

94. **Case IV.** *Given two angles and the side opposite one of them*, as A, B, a.

Pass to the polar, solve that and then pass back. Or, solve directly by means of $(144)_1$, $(171)_1$, $(171)_4$. There may be two solutions.

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95. **Case V.** *Given the three sides*, a, b, c.

Solve by means of equations (165).

---

96. **Case VI.** *Given the three angles.*

Solve by means of equations (166).

---

97. **Area of the spherical triangle.** Let A, B, C, be the angles of the triangle, R the radius of the sphere, E the spherical excess, and K the area, then

$$E = A + B + C - 180^\circ$$

$$K = \frac{E}{180^\circ}\,\pi R^2 \qquad (182)$$

EXAMPLES.

1. Given $a = 70^\circ\ 4'\ 18''$, $b = 63^\circ\ 21'\ 27''$, $c = 59^\circ\ 16'\ 23''$, to find the angles A and B.

   Ans. $A = 81^\circ\ 38'\ 20''$, $B = 70^\circ\ 9'\ 38''$.

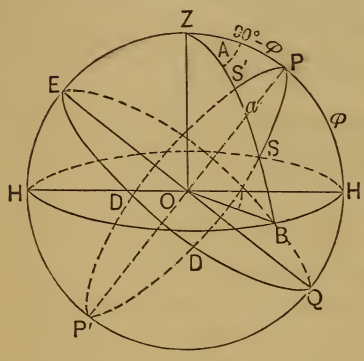
2. Given the three sides of a spherical triangle, $120^\circ\ 43'\ 37''$, $109^\circ\ 55'\ 42''$, and $116^\circ\ 38'\ 83''$; required the angles.

   Ans. $115^\circ\ 13'\ 26''$, $98^\circ\ 21'\ 40''$, $109^\circ\ 50'\ 22''$.

Applications of Spherical Trigonometry.

98. **Definitions.** *The celestial sphere* is the imaginary concave surface in which all the celestial bodies appear to be situated. Its radius is considered as infinitely long.

The sensible horizon is the circle in which a tangent plane to the surface of the earth cuts the celestial sphere.



The real horizon is the great circle in which a plane through the centre of the earth cuts the surface of the celestial sphere.

The Zenith of the observer is a point directly over his head in the surface of the celestial sphere. It is one pole of the horizon. Let O be the place of the observer, Z his zenith, then will HBH be his horizon.

The celestial equator or *equinoctial* is the great circle in which the plane of the earth's equator produced cuts the surface of the celestial sphere, as EDQ.

The axis of the earth is the imaginary line about which the earth rotates. The axis of the celestial sphere is the same line prolonged, as, POP'.

The poles of the equinoctial are the points where the earth's axis pierces the surface of the celestial sphere, as P and P'.

The ecliptic is the great circle of the celestial sphere cut by the plane of the earth's orbit.

The equinoxes are the points of intersection of the ecliptic and the celestial equator.

Hour circles or *circles of declination* are great circles passing through the poles of the equinoctial, as PSP'.

The celestial meridian of the observer is the great circle of intersection of the plane of the terrestrial meridian of the observer with the surface of the celestial sphere.

Vertical circles are great circles passing through the zenith of the observer, as ZSB.

The east and west points are where the vertical circles perpendicular to the meridian of the observer cut the horizon.

Prime verticals are verticals passing through the east and west points.

The obliquity of the ecliptic is the angle between the plane of the ecliptic and the plane of the equator, and is about 23° 27′.

The declination of a star is its distance north or south of the celestial equator. If S be the place of a star DS will be its declination. It corresponds to terrestrial latitude.

The polar distance of a star is the complement of its declination, as PS.

The right ascension of a star is the dihedral angle between the *hour circle* of the star and an established meridian. The established astronomical meridian passes through that equinox which is in the constellation Aries; and right ascension is measured from this meridian *east* through 360°. The right ascension and declination of celestial bodies are given in *nautical almanacs* or *ephemerides*.

The difference between the right ascensions of two stars is the angle between their hour circles. Thus, if S and S' are the positions of two stars, PS and PS' their hour circles, then will the difference between their right ascensions be SPS', which is measured by the arc DD on the equator between the respective hour circles.

The azimuth of a star is the angle between the meridian of the place and the vertical through the star, as HB, or HOB = the angle A at Z.

The hour angle of a star is the angle between the hour circle of the star and the meridian of the place, as ZPS. When given in degrees, it may be reduced to hours by dividing by 15, or to minutes by multiplying by 4; for the earth revolves through 15 degrees in one hour, or $\frac{1}{4}$ of a degree in one minute.

The altitude of a star is its angular elevation above the horizon, as BS.

The zenith distance of a star is the complement of its altitude, as ZS.

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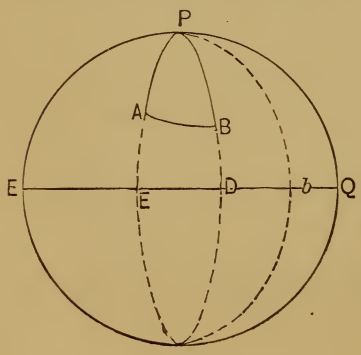
99. *To find the shortest distance between two places on the earth's surface given by their latitudes and longitudes.*



Let A and B be the places, DB the latitude of B, EA that of A, bD and bE, their respective longitudes; then in the triangle APB, we have the angle $APB = ED$ = the difference of their longitudes; PB and PA the co-latitudes, or polar distances. The arc AB will be the arc of a great circle. Hence there will be given two sides and the included angle, and may be solved by Case I., Article 91.

EXAMPLES.

1. The latitude of the government post-office at New York City is 40° 42′ 44″ N., its longitude 74° 0′ 24″ W.; the latitude of Liverpool is 53° 25′ N., and longitude 3° W., what is the shortest distance between them in miles on the earth's surface, the earth being considered a sphere whose radius is 3,956 miles.

Ans. 3,305 miles.

2. The latitude of San Francisco being 37° 48′ N., its longitude 122° 23′ W., and that of New York, that given in example 1, find the shortest distance on the earth's surface between them.

Ans. 2,562 miles.

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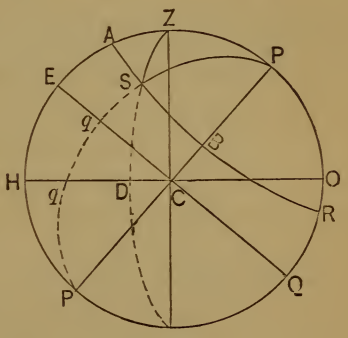
100. *The latitude of a place equals the elevation of the pole above the horizon.*



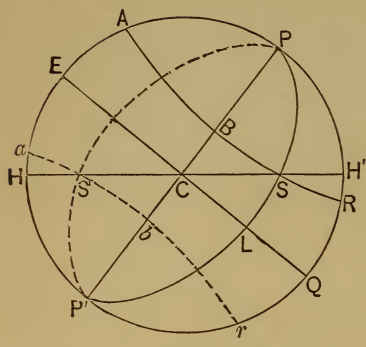
For if Z be the zenith of the observer, EZ will be his latitude; and P being the pole of the sphere, HO the horizon, we have $ECP = ZCO = 90°$. Taking ZCP from both, leaves $ECZ = PCO =$ latitude.

---

101. *To find the time of sunrise at any given place on a given day.*

Let C be the place of the observer, HCH' his horizon, RBA the small circle in which the sun appears to move, P the pole, EQ the

equator, PCH' the latitude of the observer. The sun will appear to rise when it comes into the horizon at S. Then in the triangle SLC, right angled at L, we have $SL =$ the declination of the sun, SCL = the co-latitude of the observer; hence, CL which measures the hour angle CPL, may be found from one of Napier's rules, giving



$$\sin CL = \tan Lat. \,.\, \tan Dec.$$

The sun would be six hours in describing the arc BA, and when at A it would be noon; hence CPL reduced to time, will be the time of rising *before* six o'clock. The same rule applies to any other celestial body. If rba be the path of the star, the time of rising will be six hours less the hour angle $S'Pb$.

EXAMPLES.

1. Required the time of sunrise at Hoboken on the longest day of the year; the latitude of the place being 40° 43′ 48″. The greatest declination of the sun will be 23° 27′.

$Lat. = 40° 43′ 48″$	log tan 9.935027
$Dec. = 23° 27′$	log tan 9.637265
$hour\ angle = 21° 55′ 54″$	log sin 9.572292

$= 1h.\ 27m.\ 43.6s.$ which taken from $6h.$ gives $4h.\ 32m.\ 16.4s.$

2. Required the time of sunrise in the preceding example, on the shortest day of the year.

Ans. $7h.\ 27m.\ 43.6s.$

[The declination of the sun will be 23° 27′ south; and the preceding computation will give the time *after* six o'clock.]

3. Required the length of the longest day at Hoboken.

[It will be found that the number of hours in the length of the day will be twice the hour of sunset; and hence the length of the night will be twice that of the sunrise.]

4. When the declination of the sun is 23° 27′, at what latitude will the sun not rise?

[If the declination be north it will rise at that place at midnight, or in other words be above the horizon 24 hours; but if the declination be south it will just come to the horizon at noon.]

5. At what time will the sun rise for a place on the equator?

[It will be found to be independent of the declination of the sun, and be at six o'clock.]

6. When the declination of the sun is 15° N., what will be the latitude of the place at which the sun rises at 4 o'clock?

[No allowance in these examples is made for refraction, semi-diameter of the sun, ellipsoidal form of the earth, or change in declination from mean noon.]

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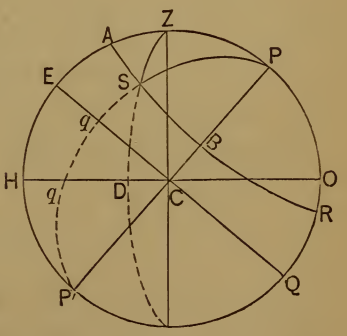
102. *To find the time of the day from the altitude of the sun.*



Let C be the place of the observer, S the sun (or star), ABR its path; then will EZ be the latitude of the observer, ZP = the co-latitude, DS = the altitude of the sun, ZS = co-altitude or zenith distance, qS = the declination of the sun, PS = co-declination or polar distance, and ZPS = the hour angle, sought, which reduced to time will be the hours before noon if S be east of the meridian, and after noon if it be west of the meridian. In the tri-

angle ZPS, three sides are known to find the angle P. Hence equation $(163)_1$ gives

$$\cos \tfrac{1}{2}ZPS = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}},$$

where

$$2s = (90^\circ - L) + (90^\circ - A) + (90^\circ - D) = 270^\circ - (L + A + D)$$

$$\therefore\ s = 135^\circ - \tfrac{1}{2}(L + A + D)$$

$$a = 90^\circ - A,\ b = 90^\circ - L,\ c = 90^\circ - D.$$

EXAMPLE.—In latitude 40° 21′ N., when the declination of the sun is 3° 20′ S., and its altitude 36° 12′, what is the time of day, after noon?

Ans. 2*h.* 17*m.* 20*s.*

---

103. *To find the azimuth at extreme elongation of a star.* If S', in the figure in the next article, be the star, then at the extreme elongation $ZS'P$ will be a right angle, PS' the polar distance of the star, and ZP the co-latitude, and by Napier's rules we have

$$\sin (90^\circ - \delta') = \sin (90^\circ - \varphi) \sin A,$$

or

$$\cos \delta' = \cos \varphi \sin A;$$

$$\therefore\ \sin A = \frac{\cos \delta'}{\cos \varphi}$$

EXAMPLE.—If $\delta' = 88^\circ\ 41'\ 24''$, $\varphi = 40^\circ\ 43'\ 48''$, required A. *Ans.* $A = 2^\circ\ 0'\ 28''$.

---

104. *Given the right ascensions of two stars and their declinations, to find their common azimuth angle when in the same vertical, and their hour angles.*

Let S and S' be the respective positions of the stars.

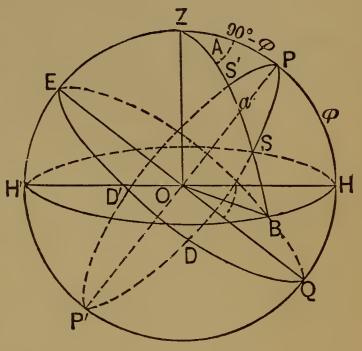
φ = the latitude of the place = EZ,

δ = the declination of $S = DS$,

δ' = the declination of $S' = D'S'$,

$\alpha - \alpha'$ = the difference of their right ascensions = SPS',

a = the side SS',



then

$ZP = 90° - \varphi$ = the co-latitude,

$PS = 90° - \delta$ = the polar distance of S,

$PS' = 90° - \delta$ = the polar distance of S',

$A = PZS = HOB$ = the azimuth.

From equation $(154)_1$

$$\cos a = \sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha - \alpha') \quad (183)$$

Let m and N be auxiliary quantities and have such values that

$$m \sin N = \sin \delta' \quad (184)$$

$$m \cos N = \cos \delta' \cos (\alpha - \alpha') \quad (185)$$

which may be done since by these two equations the two unknown quantities m and N may be found.

Dividing (184) by (185),

$$\tan N = \frac{\tan \delta'}{\cos (\alpha - \alpha')}; \quad (186)$$

also

$$m = \frac{\sin \delta'}{\sin N}, \text{ or } = \frac{\cos \delta' \cos (\alpha - \alpha')}{\cos N}. \quad (187)$$

Substituting (184) and (185) in (183) reducing and substituting m from (187) gives

$$\cos a = m \cos (\delta - N) = \frac{\sin \delta' \cos (\delta - N)}{\sin N}. \quad (188)$$

From ZPS', equation $(144)_1$,

$$\sin S' = \frac{\sin A \cos \varphi}{\cos \delta'} \quad (189)$$

and from SPS',

$$\sin S' = \frac{\cos \delta \sin (\alpha - \alpha')}{\sin a}; \quad (190)$$

dividing and reducing, gives

$$\sin A = \frac{\cos \delta \cos \delta' \sin (\alpha - \alpha')}{\cos \varphi \sin a}, \quad (191)$$

which gives the required azimuth.

To find the hour angle $ZPS' = t$

From equations (156)

$$\begin{aligned} \sin t \cot A &= \sin (90^\circ - \varphi) \cot (90^\circ - \delta) \\ &\quad - \cos (90^\circ - \varphi) \cos t \\ &= \cos \varphi \tan \delta - \sin \varphi \cos t. \end{aligned}$$

Let

$$\left.\begin{aligned} b \sin B &= \sin \varphi \\ b \cos B &= \cot A \end{aligned}\right\} \quad (192)$$

then

$$b = \frac{\sin \varphi}{\sin B}, \qquad \tan B = \frac{\sin \varphi}{\cot A}. \quad (193)$$

Substituting (192) in the equation immediately preceding, we have

$$b \sin t \cos B + b \sin B \cos t = \cos \varphi \tan \delta$$

or

$$\sin (t + B) = \cot \varphi \tan \delta \sin B, \quad (194)$$

by means of which t may be determined.

EXAMPLE.

To find the common azimuth, and the corresponding hour angle for Polaris and ε Ursæ Majoris at the place whose latitude is 40° 43′ 48″.5, and longitude 0*h*. 12*m*. 8*s*. east of the meridian of Washington. (This is very nearly the latitude and longitude of Stevens Institute, Hoboken, N. J.)

Since the right ascension and declination of the stars is continually, though very slowly, changing, it will be necessary when great accuracy is desired, to ascertain these quantities for the date of the proposed observation. Making the computation for January 1, 1884, we find from the Nautical Almanac for that date that for

Polaris the declination was..... 88° 41′ 24″.87,
and the right ascension..... 1*h*. 16*m*. 14.29*s*.
and for ε Ursæ Majoris..... dec = 56° 35′
R. A. = 12*h*. 53*m*.

The quantities for either star may be primed. Let $\delta' = 88°\ 41'\ 24''.87$, $\delta = 56°\ 35'$. Then $\alpha - \alpha'$ reduced to arc will be 174° 11′ 25″.6. The work may be arranged as follows:

			By (186) log		By (188) log		By (191) log
$\delta' =$	88° 41′ 24″.87	tan	11.640856	sin	9.999886	cos	8.359028
$\delta =$	56° 35′		.....		.....	cos	9.740334
$\alpha - \alpha =$	174° 11′ 25″.6	sec	0.002236*n*		.....	sin	9.005275
$N =$	91° 18′ 11″	tan	11.643092*n*	csc	0.000113		
$\delta - N = -$	34° 48′ 11″		.....	ccs	9.914844		
$a =$	34° 43′ 11″		.....	cos	9.916843	csc	0.244159

			By (193)		By (194)		
$\varphi =$	40° 43′ 48″.5	sin	9.814579	cot	10.064971	sec	0.120450
$A =$	0° 10′ 9″	tan	7.470148		.....	sin	7.470146
$B =$	0° 6′ 37″.3	tan	7.284727	sin	7.284727		
$\delta' =$	88° 41′ 26″.87		.....	tan	11.640859		
$t + B =$	5° 36′ 55″		.....	sin	8.990557		
$\therefore t =$	5° 30′ 18″						
	= 0*h*. 22*m*. 1*s*.						

This method may be used by surveyors for finding the true meridian, and is useful when time enters the computation, for an ordinary time piece may be used. Thus, observe when Polaris is in the same vertical with ε Ursæ Majoris (the second star in the handle of the great dipper from the bowl); then in 22 minutes the polar star will be on the meridian.

The hour angle of Polaris when it and α Ursæ Majoris have the same azimuth is $2h.\ 11m.\ 36s.$

When Polaris and δ Ursæ Majoris are in the same vertical the common azimuth is $0°\ 26'\ 28''.6$ and the corresponding hour angle of Polaris is $0h.\ 37m.\ 10.45s.$

The hour angles of α and δ Ursæ Majoris will be much less than that of Polaris when the azimuths are equal.

Solution of Oblique Triangles.

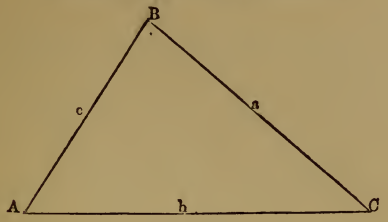


Fig. 108.

	GIVEN.	SOUGHT.	FORMULÆ.
22	A, B, a	C, b, c	$C = 180° - (A + B)$, $\quad b = \frac{a}{\sin A} \cdot \sin B$, $\quad c = \frac{a}{\sin A} \sin (A + B)$
23	A, a, b	B, C, c	$\sin B = \frac{\sin A}{a} \cdot b$, $\quad C = 180° - (A + B)$, $\quad c = \frac{a}{\sin A} \cdot \sin C$.
24	C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$
25		$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B)$
26		A, B	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$, $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$
27		c	$c = (a + b)\frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b)\frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$
28		area	$K = \frac{1}{2} a b \sin C$.
29	a, b, c	A	Let $s = \frac{1}{2}(a + b + c)$; $\sin \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}}$
30			$\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$; $\tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
31			$\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$; $\quad \text{vers } A = \frac{2(s-b)(s-c)}{bc}$
32		area	$K = \sqrt{s(s-a)(s-b)(s-c)}$
33	A, B, C, a	area	$K = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$

GENERAL FORMULÆ.

34 $\sin A = \frac{1}{\operatorname{cosec} A} = \sqrt{1-\cos^2 A} = \tan A \cos A$

35 $\sin A = 2 \sin \frac{1}{2} A \cos \frac{1}{2} A = \operatorname{vers} A \cot \frac{1}{2} A$

36 $\sin A = \sqrt{\frac{1}{2} \operatorname{vers} 2A} = \sqrt{\frac{1}{2}(1-\cos 2A)}$

37 $\cos A = \frac{1}{\sec A} = \sqrt{1-\sin^2 A} = \cot A \sin A$

38 $\cos A = 1 - \operatorname{vers} A = 2\cos^2 \frac{1}{2} A - 1 = 1 - 2\sin^2 \frac{1}{2} A$

39 $\cos A = \cos^2 \frac{1}{2} A - \sin^2 \frac{1}{2} A = \sqrt{\frac{1}{2} + \frac{1}{2} \cos 2A}$

40 $\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1}$

41 $\tan A = \sqrt{\frac{1}{\cos^2 A} - 1} = \frac{\sqrt{1-\cos^2 A}}{\cos A} = \frac{\sin 2A}{1+\cos 2A}$

42 $\tan A = \frac{1-\cos 2A}{\sin 2A} = \frac{\operatorname{vers} 2A}{\sin 2A} = \operatorname{exsec} A \cot \frac{1}{2} A$

43 $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \sqrt{\operatorname{cosec}^2 A - 1}$

44 $\cot A = \frac{\sin 2A}{1-\cos 2A} = \frac{\sin 2A}{\operatorname{vers} 2A} = \frac{1+\cos 2A}{\sin 2A}$

45 $\cot A = \frac{\tan \frac{1}{2} A}{\operatorname{exsec} A}$

46 $\operatorname{vers} A = 1 - \cos A = \sin A \tan \frac{1}{2} A = 2\sin^2 \frac{1}{2} A$

47 $\operatorname{vers} A = \operatorname{exsec} A \cos A$

48 $\operatorname{exsec} A = \sec A - 1 = \tan A \tan \frac{1}{2} A = \frac{\operatorname{vers} A}{\cos A}$

49 $\sin \frac{1}{2} A = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{\operatorname{vers} A}{2}}$

50 $\sin 2A = 2 \sin A \cos A$

51 $\cos \frac{1}{2} A = \sqrt{\frac{1+\cos A}{2}}$

52 $\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

GENERAL FORMULÆ.

53. $\tan \frac{1}{2} A = \frac{\tan A}{1 + \sec A} = \operatorname{cosec} A - \cot A = \frac{1 - \cos A}{\sin A} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$

54. $\tan 2 A = \frac{2 \tan A}{1 - \tan^2 A}$

55. $\cot \frac{1}{2} A = \frac{\sin A}{\operatorname{vers} A} = \frac{1 + \cos A}{\sin A} = \frac{1}{\operatorname{cosec} A - \cot A}$

56. $\cot 2 A = \frac{\cot^2 A - 1}{2 \cot A}$

57. $\operatorname{vers} \frac{1}{2} A = \frac{\frac{1}{2} \operatorname{vers} A}{1 + \sqrt{1 - \frac{1}{2} \operatorname{vers} A}} = \frac{1 - \cos A}{2 + \sqrt{2 (1 + \cos A)}}$

58. $\operatorname{vers} 2 A = 2 \sin^2 A$

59. $\operatorname{exsec} \frac{1}{2} A = \frac{1 - \cos A}{(1 + \cos A) + \sqrt{2 (1 + \cos A)}}$

60. $\operatorname{exsec} 2 A = \frac{\tan^2 A}{1 - \tan^2 A}$

61. $\sin (A \pm B) = \sin A \,.\, \cos B \pm \sin B \,.\, \cos A$

62. $\cos (A \pm B) = \cos A \,.\, \cos B \mp \sin A \,.\, \sin B$

63. $\sin A + \sin B = 2 \sin \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

64. $\sin A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$

65. $\cos A + \cos B = 2 \cos \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

66. $\cos B - \cos A = 2 \sin \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$

67. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A + B) \sin (A - B)$

68. $\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$

69. $\tan A + \tan B = \frac{\sin (A + B)}{\cos A \,.\, \cos B}$

70. $\tan A - \tan B = \frac{\sin (A - B)}{\cos A \,.\, \cos B}$

EXPLANATION

OF

THE TABLES.

The arithmetical complement of a number is the remainder after subtracting it from 10. It is often used to avoid the necessity of subtracting one logarithm from another. Instead of subtracting a logarithm, we may *add* its arithmetical complement and subtract 10 from the result. Thus $a - b$ is the same as $a + (10 - b) - 10$.

The arithmetical complement is indicated by the abbreviation "ar co," or "a. c." Thus to find the logarithm of

$$\frac{\sin 50^\circ}{\cos 60^\circ},$$

we have

50°	log sin	9.884254
60°	log cos	9.698970
subtracting,		0.185284

or,

50°	log sin	9.884254
60°	ar co log cos	0.301030
adding,		10.185284
subtracting		10
gives		0.185284,

which is the same as the preceding result.

The logarithm of a number consists of two parts, a whole number called the *characteristic*, and a decimal called the *mantissa*. All numbers which consist of the same figures standing in the same order have the same mantissa, regardless of the position of the decimal point in the number, or of the number of ciphers which precede or follow the significant figures of the number. The value of the characteristic depends entirely on the position of the decimal point in the number. It is always one less than the number of

figures in the number to the left of the decimal point. The value is therefore diminished by one every time the decimal point of the number is removed one place to the left, and *vice versa*. Thus

Number.	*Logarithm.*
13840.	4.141136
1384.0	3.141136
138.40	2.141136
13.84	1.141136
1.384	0.141136
.1384	$\bar{1}$.141136
.01384	$\bar{2}$.141136
.001384	$\bar{3}$.141136
etc.	etc.

The mantissa is always positive even when the characteristic is negative. We may avoid the use of a negative characteristic by arbitrarily adding 10, which may be neglected at the close of the calculation. By this rule we have

Number.	*Logarithm.*
1.384	0.141136
.1384	9.141136
.01384	8.141136
.001384	7.141136
etc.	etc.

No confusion need arise from this method in finding a number from its logarithm; for although the logarithm 6.141136 represents either the number 1,384,000, or the decimal .0001384, yet these are so diverse in their values that we can never be uncertain in a given problem which to adopt.

The first table contains the mantissas of logarithms, carried to six places of decimals, for numbers between 1 and 9999, inclusive. The first three figures of a number are given in the first column, the fourth at the top of the other columns. The first two figures of the mantissa are given only in the second column, but these are understood to apply to the remaining four figures in either column following, which are comprised between the same horizontal lines with the two.

If a number (after cutting off the ciphers at either end) consists of not more than four figures, the mantissa may be taken direct from the table; but by interpolation the logarithm of a number having six figures may be obtained. The last column contains the average difference of consecutive logarithms on

the same line, but for a given case the difference needs to be verified by actual subtraction, at least so far as the last figure is concerned. The lower part of the page contains a complete list of differences, with their multiples divided by 10.

To find the logarithm of a number having six figures :—Take out the mantissa for the four superior places directly from the table, and find the difference between this mantissa and the next greater in the table. Add to the mantissa taken out the quantity found in the table of proportional parts, opposite the difference, and in the column headed by the fifth figure of the number; also add $\frac{1}{10}$ the quantity in the column headed by the sixth figure. The sum is the mantissa required, to which must be prefixed a decimal point and the proper characteristic.

Example.—Find the log of 23.4275.

For 2342 mantissa is	369587
" diff. 185 col. 7	129.5
" " " " 5	9.2

Ans. For 23.4275 log is 1.369726

The decimals of the corrections are added together to determine the nearest value of the sixth figure of the mantissa.

To find the number corresponding to a given logarithm.—If the given mantissa is not in the table find the one next less, and take out the four figures corresponding to it; divide the difference between the two mantissas by the tabular difference in that part of the table, and annex the figures of the quotient to the four figures already taken out. Finally, place the decimal point according to the rule for characteristics, prefixing or annexing ciphers if necessary. The division required is facilitated by the table of proportional parts, which furnishes by inspection the figures of the quotient.

Example.—Find the number of which the logarithm is 8.263927

		8.263927
First 4 figures 1836 from		263873
	Diff.	54.0
Tabular diff. = 236	∴ 5th fig. = 2	47.2
		6.80
	6th fig. = 3	7.08

Ans. No. = .183,623,000.

The number derived from a six-place logarithm is not reliable beyond the sixth figure.

At the end of the first table is a small table of logarithms of numbers from 1 to 100, with the characteristic prefixed, for easy reference when the given number does not exceed two digits. But the same mantissas may be found in the larger table.

The logarithmic sine, tangent, etc. of an arc is the logarithm of the natural sine, tangent, etc. of the same arc, but with 10 added to the characteristic to avoid negatives. This table gives log sines, tangents, cosines, and cotangents for every minute of the quadrant. With the number of degrees at the left side of the page are to be read the minutes in the left-hand column ; with the degrees on the right-hand side are to be read the minutes in the right-hand column. When the degrees appear at the top of the page the top headings must be observed, when at the bottom those at the bottom. Since the values found for arcs in the first quadrant are duplicated in the second, the degrees are given from 0° to 180°. The differences in the logarithms due to a change of one second in the arc are given in adjoining columns.

To find the log. sin, cos, tan, or cot of a given arc.: Take out from the proper column of the table the logarithm corresponding to the given number of degrees and minutes. If there be any seconds multiply them by the adjoining tabular difference, and apply their product as a correction to the logarithm already taken out. The correction is to be *added* if the logarithms of the table are increasing with the angle, or *subtracted* if they are decreasing as the angle increases. In the first quadrant the log sines and tangents increase, and the log. cosines and cotangents decrease as the angle increases.

Example.—Find the log sin of 9° 28′ 20″.

Log sin of 9° 28′ is	9.216097
Add correction 20 × 12.62	252
Ans.	9.216349

Example.—Find the log cot of 9° 28′ 20″.

Log cotan of 9° 28′ is	10.777948
Subtract correction 20 × 12.97	259
Ans.	10.777689

To find the angle or arc corresponding to a given logarithmic sine, tangent, cosine, or cotangent.—If the given logarithm is found in the proper column take out the degrees and minutes directly; if not, find the two consecutive logarithms between which the given logarithm would fall, and adopt that one which corresponds to the least number of minutes; which minutes take out with the degrees, and divide the difference between this logarithm and the given one by the adjoining tabular difference for a quotient, which will be the required number of seconds.

With logarithms to six places of decimals the quotient is not reliable beyond the tenth of a second.

Example.—9.383731 is the log tan of what angle?

Next *less* 9.383682 gives		13° 36′
Diff.	49.00 ÷ 9.20 =	05″.3
	Ans.	13° 36′ 05″.3

Example.—9.249348 is the log cos of what angle?

Next *greater* 583 gives		79° 46′
Diff.	235 ÷ 11.67 =	20″.1
	Ans.	79° 46′ 20″.1

The above rules do not apply to the first two pages of this table (except for the column headed cosine at top) because here the differences vary so rapidly that interpolation made by them in the usual way will not give exact results.

On the first two pages, the *first* column contains the number of seconds for every minute from 1′ to 2°; the minutes are given in the *second*, the log. sin. in the *third*, and in the *fourth* are the last three figures of a logarithm which is the difference between the log sin and the logarithm of the number of seconds in the first column. The first three figures and the characteristic of this logarithm are placed, once for all, at the head of the column.

To find the log sin of an arc less than 2° given to seconds.—Reduce the given arc to seconds, and take the logarithm of the number of seconds from the table of logarithms, and *add* to this the logarithm from the fourth column opposite the same number of seconds. The sum is the log sin required.

The logarithm in the fourth column may need a slight inter-

polation of the last figure, to make it correspond closely to the given number of seconds.

Example.—Find the log sin of 1° 39′ 14″.4.

1° 39′ 14″.4 = 5954″.4	log 3.774838
	add $(q - l)$ 4.685515
	Ans. log sin 8.460353

Log tangents of small arcs are found in the same way, only taking the last four figures of $(q - l)$ from the *fifth* column.

Example.—Find the log tan of 0° 52′ 35″.

52′ 35″ = (3120″ + 35″) = 3155″	log 3.498999
	add $(q - l)$ 4.685609
	Ans. log tan 8.184608

To find the log cotangent of an angle less than 2° given to seconds.—Take from the column headed $(q + l)$ the logarithm corresponding to the given angle, interpolating for the last figure if necessary, and from this *subtract* the logarithm of the number of seconds in the given angle.

Example.—Find the log cotan of 1° 44′ 22″.5.

	$q + l$ 15.314292
6240″ + 22″.5 = 6262.5	log 3.796748
	Ans. 11.517544

These two pages may be used in the same way when the given angle lies between 88° and 92°, or between 178° and 180°; but if the number of degrees be found at the *bottom* of the page, the title of each column will be found there also; and if the number of degrees be found on the *right hand* side of the page, the number of minutes must be found in the right hand column, and since here the minutes increase upward, the number of seconds on the same line in the first column must be *diminished* by the odd seconds in the given angle to obtain the number whose logarithm is to be used with $(q \pm l)$ taken from the table.

Example.—Find the log cos of 88° 41′ 12″.5

	$(q - l)$ 4.685537
4740″ − 12″.5 = 4727.5	log 3.674631
	Ans. 8.360168

Example.—Find the log tan of 90° 30′ 50″.

$1800'' + 50'' = 1850''$

$q + l$	15.314413
log	3.267172
Ans.	12.047241

To find the arc corresponding to a given log sin, cos, tan, or cotan which falls within the limits of the first two pages of the table.

Find in the proper column two consecutive logarithms between which the given logarithm falls. If the title of the given function is found at the top of that column read the degrees from the top of the page; if at the bottom read from the bottom.

Find the value of $(q - l)$ or $(q + l)$, as the case may require, corresponding to the given log (interpolating for the last figure if necessary). Then if q = given log and l = log of number of seconds, n, in the required arc, we have at once $l = q - (q - l)$ or $l = (q + l) - q$, whence n is easily found.

Find in the first column two consecutive quantities between which the number n falls, and if the degrees are read from the *left hand* side of the page, adopt the *less*, take out the minutes from the second column, and take for the seconds the difference between the quantity adopted and the number n. But if the degrees are read from the *right hand* side of the page, adopt the *greater* quantity, take out the minutes on the same line from the right-hand column, and for the seconds take the difference between the number adopted and the number n.

Example.—11.734268 is the log cot of what arc?

$q + l$		15.314376
q		11.734268
∴ n =	3802.8	3.580108
For 1° adopt	3780.	giving 03′
Difference	22″.8	

Ans. 1° 03′ 22″.8 or 178° 56′ 37″.2.

Example.—8.201795 is the log cos of what arc?

$q - l$		4.685556
q		8.201795
∴ n =	3282″.8	3.516239
For 89° adopt	3300.	giving 05′
Difference	17″.2	

Ans. 89° 05′ 17″.2 or 90° 54′ 42″.8.

LOGARITHMS OF NUMBERS.

No. 100 L. 000.] [No. 109 L. 040.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876							
					0300	0724	1147	1570	1993	2415	424
3	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947			
									0361	0775	416
5	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789									
			0195	0600	1004	1408	1812	2216	2619	3021	404
8	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811				
	04							0207	0602	0998	397

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
433	43.3	86.6	129.9	173.2	216.5	259.8	303.1	346.4	389.7
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
431	43.1	86.2	129.3	172.4	215.5	258.6	301.7	344.8	387.9
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
429	42.9	85.8	128.7	171.6	214.5	257.4	300.3	343.2	386.1
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
427	42.7	85.4	128.1	170.8	213.5	256.2	298.9	341.6	384.3
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
425	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
423	42.3	84.6	126.9	169.2	211.5	253.8	296.1	338.4	380.7
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
421	42.1	84.2	126.3	168.4	210.5	252.6	294.7	336.8	378.9
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
419	41.9	83.8	125.7	167.6	209.5	251.4	293.3	335.2	377.1
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
417	41.7	83.4	125.1	166.8	208.5	250.2	291.9	333.6	375.3
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
415	41.5	83.0	124.5	166.0	207.5	249.0	290.5	332.0	373.5
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
413	41.3	82.6	123.9	165.2	206.5	247.8	289.1	330.4	371.7
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
411	41.1	82.2	123.3	164.4	205.5	246.6	287.7	328.8	369.9
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
409	40.9	81.8	122.7	163.6	204.5	245.4	286.3	327.2	368.1
408	40 8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
407	40.7	81.4	122.1	162.8	203.5	244.2	284.9	325.6	366.3
406	40.6	81.2	121.8	162.4	203.0	243 6	284.2	324.8	365.4
405	40.5	81.0	121.5	162.0	202.5	243.0	283.5	324.0	364.5
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
403	40.3	80.6	120.9	161.2	201.5	241.8	282.1	322.4	362.7
402	40.2	80.4	120.6	160.8	201.0	241 2	281.4	321.6	361.8
401	40.1	80.2	120.3	160.4	200.5	240.6	280.7	320.8	360.9
400	40.0	80·0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
399	39.9	79.8	119.7	159.6	199.5	239.4	279.3	319.2	359.1
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
397	39.7	79.4	119.1	158.8	198.5	238.2	277.9	317.6	357.3
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	356.4
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316 0	355.5

No. 110 L. 041.] [No. 119 L. 078.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993								
				0380	0766	1153	1538	1924	2309	2694	386
3	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942		
										0320	379
5	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668						
						0038	0407	0776	1145	1514	370
8	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5
394	39.4	78.8	118.2	157.6	197 0	236.4	275.8	315.2	354.6
393	39.3	78.6	117.9	157.2	196.5	235.8	275.1	314.4	353.7
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
391	39.1	78.2	117.3	156.4	195.5	234.6	273.7	312.8	351.9
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
389	38.9	77.8	116.7	155.6	194.5	233.4	272.3	311.2	350.1
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
387	38.7	77.4	116.1	154.8	193.5	232.2	270.9	309.6	348.3
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
385	38.5	77.0	115.5	154.0	192.5	231.0	269.5	308.0	346.5
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
383	38.3	76.6	114.9	153.2	191.5	229.8	268.1	306.4	344.7
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
381	38.1	76.2	114.3	152.4	190.5	228.6	266.7	304.8	342.9
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
379	37.9	75.8	113.7	151.6	189.5	227.4	265.3	303.2	341.1
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
377	37.7	75.4	113.1	150.8	188.5	226.2	263.9	301.6	339.3
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
375	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
373	37.3	74.6	111.9	149.2	186.5	223.8	261.1	298.4	335.7
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
371	37.1	74.2	111.3	148.4	185.5	222.6	259.7	296.8	333.9
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
369	36.9	73.8	110.7	147.6	184.5	221.4	258.3	295.2	332.1
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
367	36.7	73.4	110.1	146.8	183.5	220.2	256.9	293.6	330.3
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
365	36.5	73.0	109.5	146.0	182.5	219.0	255.7	292.0	328.5
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
363	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
361	36.1	72.2	108.3	144.4	180.5	216.6	252.7	288.8	324.9
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0
359	35.9	71.8	107.7	143.6	179.5	215.4	251.3	287.2	323.1
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
357	35.7	71.4	107.1	142.8	178.5	214.2	249.9	285.6	321.3
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4

No. 120 L. 079.] [No. 134 L. 130.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
120	079181	9543	9904	0266	0626	0987	1347	1707	2067	2426	360
1	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	0026	346
6	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	0253	338
9	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245	330
2	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	0012	323

13

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
355	35.5	71.0	106.5	142.0	177.5	213.0	248.5	284.0	319.5
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
353	35.3	70.6	105.9	141.2	176.5	211.8	247.1	282.4	317.7
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
351	35.1	70.2	105.3	140.4	175.5	210.6	245.7	280.8	315.9
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
349	34.9	69.8	104.7	139.6	174.5	209.4	244.3	279.2	314.1
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
347	34.7	69.4	104.1	138.8	173.5	208.2	242.9	277.6	312.3
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
345	34.5	69.0	103.5	138.0	172.5	207.0	241.5	276.0	310.5
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
343	34.3	68.6	102.9	137.2	171.5	205.8	240.1	274.4	308.7
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
341	34.1	68.2	102.3	136.4	170.5	204.6	238.7	272.8	306.9
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
339	33.9	67.8	101.7	135.6	169.5	203.4	237.3	271.2	305.1
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
337	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
335	33.5	67.0	100.5	134.0	167.5	201.0	234.5	268.0	301.5
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
333	33.3	66.6	99.9	133.2	166.5	199.8	233.1	266.4	299.7
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
331	33.1	66.2	99.3	132.4	165.5	198.6	231.7	264.8	297.9
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
329	32.9	65.8	98.7	131.6	164.5	197.4	230.3	263.2	296.1
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
327	32.7	65.4	98.1	130.8	163.5	196.2	228.9	261.6	294.3
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
325	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
323	32.3	64.6	96.9	129.2	161.5	193.8	226.1	258.4	290.7
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8

No. 135 L. 130.] [No. 149 L. 175.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879										
		0194	0508	0822	1136	1450	1763	2076	2389	2702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
1	9219	9527	9835								
				0142	0449	0756	1063	1370	1676	1982	307
2	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868					
							0168	0469	0769	1068	301
5	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
321	32.1	64.2	96.3	128.4	160.5	192.6	224.7	256.8	288.9
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
319	31.9	63.8	95.7	127.6	159.5	191.4	223.3	255.2	287.1
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	254.4	286.2
317	31.7	63.4	95.1	126.8	158.5	190.2	221.9	253.6	285.3
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
315	31.5	63.0	94.5	126.0	157.5	189.0	220.5	252.0	283.5
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
313	31.3	62.6	93.9	125.2	156.5	187.8	219.1	250.4	281.7
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
311	31.1	62.2	93.3	124.4	155.5	186.6	217.7	248.8	279.9
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
309	30.9	61.8	92.7	123.6	154.5	185.4	216.3	247.2	278.1
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
307	30.7	61.4	92.1	122.8	153.5	184.2	214.9	245.6	276.3
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
305	30.5	61.0	91.5	122.0	152.5	183.0	213.5	244.0	274.5
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
303	30.3	60.6	90.9	121.2	151.5	181.8	212.1	242.4	272.7
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
301	30.1	60.2	90.3	120.4	150.5	180.6	210.7	240.8	270.9
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
299	29.9	59.8	89.7	119.6	149.5	179.4	209.3	239.2	269.1
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
297	29.7	59.4	89.1	118.8	148.5	178.2	207.9	237.6	267.3
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
295	29.5	59.0	88.5	118.0	147.5	177.0	206.5	236.0	265.5
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
293	29.3	58.6	87.9	117.2	146.5	175.8	205.1	234.4	263.7
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
291	29.1	58.2	87.3	116.4	145.5	174.6	203.7	232.8	261.9
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
289	28.9	57.8	86.7	115.6	144.5	173.4	202.3	231.2	260.1
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
287	28.7	57.4	86.1	114.8	143.5	172.2	200.9	229.6	258.3
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4

No. 150 L. 176.] [No. 169 L. 230.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
1	8977	9264	9552	9839							
					0126	0413	0699	0986	1272	1558	287
2	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771		
										0051	281
5	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755						
						0029	0303	0577	0850	1124	274
9	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783									
			0051	0319	0586	0853	1121	1388	1654	1921	267
3	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938		
	23									0193	256

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
285	28.5	57.0	85.5	114.0	142.5	171.0	199.5	228.0	256.5
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
283	28.3	56.6	84.9	113.2	141 5	169.8	198.1	226.4	254.7
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
281	28.1	56.2	84.3	112 4	140.5	168.6	196.7	224.8	252.9
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0
279	27.9	55.8	83.7	111.6	139.5	167.4	195.3	223.2	251.1
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
277	27.7	55.4	83.1	110.8	138.5	166.2	193.9	221.6	249.3
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
275	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	247.5
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
273	27.3	54.6	81.9	109.2	136.5	163.8	191.1	218.4	245.7
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
271	27.1	54.2	81.3	108.4	135.5	162.6	189.7	216.8	243.9
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
269	26.9	53.8	80.7	107.6	134.5	161.4	188.3	215.2	242.1
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
267	26.7	53.4	80.1	106.8	133.5	160.2	186.9	213.6	240.3
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
265	26.5	53.0	79.5	106.0	132.5	159.0	185.5	212.0	238.5
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
263	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
261	26.1	52.2	78.3	104.4	130.5	156.6	182.7	208.8	234.9
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
259	25.9	51.8	77.7	103.6	129.5	155.4	181.3	207.2	233.1
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
257	25.7	51.4	77.1	102.8	128.5	154.2	179.9	205.6	231.3
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5

No. 170 L. 230.] [No. 189 L. 278.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800			
									0050	0300	250
4	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932		
										0176	245
8	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980								
				0213	0446	0679	0912	1144	1377	1609	233
7	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
253	25.3	50.6	75.9	101.2	126.5	151.8	177.1	202.4	227.7
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
251	25.1	50.2	75.3	100.4	125.5	150.6	175.7	200.8	225.9
250	25 0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
249	24.9	49.8	74.7	99.6	124.5	149.4	174.3	199.2	224.1
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
247	24.7	49.4	74.1	98.8	123.5	148.2	172.9	197.6	222.3
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
245	24.5	49.0	73.5	98.0	122.5	147.0	171.5	196.0	220.5
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
243	24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
241	24.1	48.2	72.3	96.4	120.5	144.6	168.7	192.8	216.9
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
239	23.9	47.8	71.7	95.6	119.5	143.4	167.3	191.2	215.1
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
237	23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
235	23.5	47.0	70.5	94.0	117.5	141.0	164.5	188.0	211.5
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
233	23.3	46.6	69.9	93.2	116.5	139.8	163.1	186.4	209.7
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
231	23.1	46.2	69.3	92.4	115.5	138.6	161.7	184.8	207.9
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0
229	22.9	45.8	68.7	91.6	114.5	137.4	160.3	183.2	206.1
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205 2
227	22.7	45.4	68.1	90.8	113.5	136.2	158.9	181.6	204.3
226	22.6	45.2	67.8	90.4	113.0	135.6	158 2	180.8	203.4

No. 190 L. 278.] [No. 214 L. 332.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
190	278754	8932	9211	9489	9667	9895	0123	0351	0578	0806	228
1	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	0161	0378	0595	0813	218
200	301030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542	212
5	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	0008	0211	203
4	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
225	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
223	22.3	44.6	66.9	89.2	111.5	133.8	156.1	178.4	200.7
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
221	22.1	44.2	66.3	88.4	110.5	132.6	154.7	176.8	198.9
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
219	21.9	43.8	65.7	87.6	109.5	131.4	153.3	175.2	197.1
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
217	21.7	43.4	65.1	86.8	108.5	130.2	151.9	173.6	195.3
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
215	21.5	43.0	64.5	86.0	107.5	129.0	150.5	172.0	193.5
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
213	21.3	42.6	63.9	85.2	106.5	127.8	149.1	170.4	191.7
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
211	21.1	42.2	63.3	84.4	105.5	126.6	147.7	168.8	189.9
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
209	20.9	41.8	62.7	83.6	104.5	125.4	146.3	167.2	188.1
208	20.8	41.6	62 4	83.2	104.0	124.8	145.6	166 4	187.2
207	20.7	41.4	62.1	82.8	103.5	124.2	144.9	165.6	186.3
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
205	20.5	41.0	61.5	82.0	102.5	123.0	143.5	164.0	184.5
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
203	20.3	40.6	60.9	81.2	101.5	121.8	142.1	162.4	182.7
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8

No. 215 L. 332.] [No. 239 L. 380.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
215	332438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849			
									0047	0246	199
9	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860		
										0054	194
4	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835										
		0025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	361728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958						
						0143	0328	0513	0698	0883	185
5	371063	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849		
	38									0030	181

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
201	20.1	40.2	60.3	80.4	100.5	120.6	140.7	160.8	180.9
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
199	19.9	39.8	59.7	79.6	99.5	119.4	139.3	159.2	179.1
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
197	19.7	39.4	59.1	78.8	98.5	118.2	137.9	157.6	177.3
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
195	19.5	39.0	58.5	78.0	97.5	117.0	136.5	156.0	175.5
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
193	19.3	38.6	57.9	77.2	96.5	115.8	135.1	154.4	173.7
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
191	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.9
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
189	18.9	37.8	56.7	75.6	94.5	113.4	132.3	151.2	170.1
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
187	18.7	37.4	56.1	74.8	93.5	112.2	130.9	149.6	168.3
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
185	18.5	37.0	55.5	74.0	92.5	111.0	129.5	148.0	166.5
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
183	18.3	36.6	54.9	73.2	91.5	109.8	128.1	146.4	164.7
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
181	18.1	36.2	54.3	72.4	90.5	108.6	126.7	144.8	162.9
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
179	17.9	35.8	53.7	71.6	89.5	107.4	125.3	143.2	161.1

No. 240 L. 380.]										[No. 269 L. 431.	
N.	0	1	2	3	4	5	6	7	8	9	Diff.
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875						
						0051	0228	0405	0582	0759	177
6	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
1	9674	9847									
			0020	0192	0365	0538	0711	0883	1056	1228	173
2	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933										
		0102	0271	0440	0609	0777	0946	1114	1283	1451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956										
		0121	0286	0451	0616	0781	0945	1110	1275	1439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914									
	43		0075	0236	0398	0559	0720	0881	1042	1203	161

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
177	17.7	35.4	53.1	70.8	88.5	106.2	123.9	141.6	159.3
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
175	17.5	35.0	52.5	70.0	7.5	105.0	122.5	140.0	157.5
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
173	17.3	34.6	51.9	69.2	86.5	103.8	121.1	138.4	155.7
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
171	17.1	34.2	51.3	68.4	85.5	102.6	119.7	136.8	153.9
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
169	16.9	33.8	50.7	67.6	84.5	101.4	118.3	135.2	152.1
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
167	16.7	33.4	50.1	66.8	83.5	100.2	116.9	133.6	150.3
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
165	16.5	33.0	49.5	66.0	82.5	99.0	115.5	132.0	148.5
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
163	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.7
162	16.2	32.4	48.5	64.8	81.0	97.2	113.4	129.6	145.8
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9

No. 270 L. 431.] [No. 299 L. 476.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964						
						0122	0279	0437	0594	0752	158
6	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941		
										0095	154
2	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995						
						0146	0296	0447	0597	0748	151
9	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969									
			0116	0263	0410	0557	0704	0851	0998	1145	147
6	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
159	15.9	31.8	47.7	63.6	79.5	95.4	111.3	127.2	143.1
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
157	15.7	31.4	47.1	62.8	78.5	94.2	109.9	125.6	141.3
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4
155	15.5	31.0	46.5	62.0	77.5	93.0	108.5	124.0	139.5
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
153	15.3	30.6	45.9	61.2	76.5	91.8	107.1	122.4	137.7
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
151	15.1	30.2	45.3	60.4	75.5	90.6	105.7	120.8	135.9
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
149	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
147	14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132.3
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
145	14.5	29.0	43.5	58.0	72.5	87.0	101.5	116.0	130.5
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
143	14.3	28.6	42.9	57.2	71.5	85.8	100.1	114.4	128.7
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
141	14.1	28.2	42.3	56.4	70.5	84.6	98.7	112.8	126.9
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.0

No. 300 L. 477.] [No. 339 L. 531.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958										
		0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962								
				0099	0236	0374	0511	0648	0785	0922	137
7	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874					
							0009	0143	0277	0411	134
4	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
1	9828	9959									
			0090	0221	0353	0484	0615	0745	0876	1007	131
2	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943		
										0072	128
9	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
139	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.2	125.1
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
137	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
135	13.5	27.0	40.5	54.0	67.5	81.0	94.5	108.0	121.5
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
133	13.3	26.6	39.9	53.2	66.5	79.8	93.1	106.4	119.7
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
131	13.1	26.2	39.3	52.4	65.5	78.6	91.7	104.8	117.9
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
129	12.9	25.8	38.7	51.6	64.5	77.4	90.3	103.2	116.1
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12 7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3

No. 340 L. 531.] [No. 379 L. 579.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954			
									0079	0204	125
7	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984		
										0106	123
5	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907										
		0026	0146	0265	0385	0504	0624	0743	0863	0982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
1	9374	9491	9608	9725	9842	9959					
							0076	0193	0309	0426	117
2	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
125	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
123	12.3	24.6	36.9	49.2	61.5	73.8	86.1	98.4	110.7
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
121	12.1	24.2	36.3	48.4	60,5	72.6	84.7	96.8	108.9
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
119	11.9	23.8	35.7	47.6	59.5	71.4	83.3	95.2	107.1

No. 380. L. 579.] [No. 414 L. 617.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	579784	9898	0012	0126	0241	0355	0469	0583	0697	0811	114
1	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
9	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953	
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	
8	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864	109
9	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
7	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554	
8	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
117	11.7	23.4	35.1	46.8	58.5	70.2	81.9	93.6	105.3
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
115	11.5	23.0	34.5	46.0	57.5	69.0	80.5	92.0	103.5
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6
113	11.3	22.6	33.9	45.2	56.5	67.8	79.1	90.4	101.7
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
111	11.1	22.2	33.3	44.4	55.5	66.6	77.7	88.8	99.9
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
109	10.9	21.8	32.7	43.6	54.5	65.4	76.3	87.2	98.1
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
107	10.7	21.4	32.1	42.8	53.5	64.2	74.9	85.6	96.3
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6

No. 415 L. 618.] [No. 459 L. 662

N.	0	1	2	3	4	5	6	7	8	9	Diff.
415	618048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	0032	
7	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	0021	0123	0224	0326	
7	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342	
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	
6	9486	9586	9686	9785	9885	9984	0084	0183	0283	0382	
7	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375	
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	
6	9335	9432	9530	9627	9724	9821	9919	0016	0113	0210	
7	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181	
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	
7	9916	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72 8	83.2	93.6
103	10.3	20.6	30.9	41.2	51.5	61.8	72 1	82.4	92.7
102	10.2	20.4	30.6	40.8	51.0	61.2	71 4	81.6	91.8
101	10.1	20.2	30.3	40.4	50.5	60.6	70 7	80.8	90.9
100	10.0	20.0	30.0	40.0	50.0	60.0	70 0	80.0	90.0
99	9.9	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.1

No. 460 L. 662.] [No. 499 L. 698.

N	0	1	2	3	4	5	6	7	8	9	Diff.
460	662758	2852	2947	3041	3135	3230	3324	3418	3512	3607	
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	
7	9317	9410	9503	9596	9689	9782	9875	9967			
									0060	0153	93
8	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080	
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973				
									0063	0154	0245	
9	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151	
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	
9	9309	9398	9486	9575	9664	9753	9841	9930			
									0019	0107	
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993	
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	
9	8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	87

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
97	9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
95	9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
93	9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
91	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
89	8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	80.1
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4

No. 500 L. 698.] [No. 544 L. 736.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
500	698970	9057	9144	9231	9317	9404	9491	9578	9664	9751	
1	9838	9924									
			0011	0098	0184	0271	0358	0444	0531	0617	
2	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482	
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948		
										0033	
3	710117	0202	0287	0371	0456	0540	0625	0710	0794	0879	
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994		
										0077	
5	720159	0242	0325	0407	0490	0573	0655	0738	0821	0903	
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974										
			0055	0136	0217	0298	0378	0459	0540	0621	0702	
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
85	8.5	17.0	25.5	34.0	42.5	51.0	59.5	68.0	76.5
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6

No. 545 L. 736.] [No. 584 L. 767.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
545	736397	6476	6556	6635	6715	6795	6874	6954	7034	7113	
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	
9	9572	9651	9731	9810	9889	9968					
							0047	0126	0205	0284	79
550	740363	0442	0521	0600	0678	0757	0836	0915	0994	1073	
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	
2	9736	9814	9891	9968							
					0045	0123	0200	0277	0354	0431	
3	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	
5	9668	9743	9819	9894	9970						
						0045	0121	0196	0272	0347	
6	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101	
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.3
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6

No. 585 L. 767.] [No. 629 L. 799.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
585	767156	7230	7304	7379	7453	7527	7601	7675	7749	7823	
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	0042	
9	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778	
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	
2	9596	9669	9741	9813	9885	9957	0029	0101	0173	9245	
3	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	
6	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215	
7	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918	
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
73	7.3	14.6	21.9	29.2	36.5	43.8	51.1	58.4	65.7
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
71	7.1	14.2	21.3	28.4	35.5	42.6	49.7	56.8	63.9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
69	6.9	13.8	20.7	27.6	34.5	41.4	48.3	55.2	62.1

No. 630 L. 799.] [No. 674 L. 829.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
630	799341	9409	9478	9547	9616	9685	9754	9823	9892	9961	
1	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648	
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	
640	806180	6248	6316	6384	6451	6519	6587	6655	6723	6790	
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	
5	9560	9627	9694	9762	9829	9896	9964				
								0031	0098	0165	
6	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837	
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	
660	9544	9610	9676	9741	9807	9873	9939				
								0004	0070	0136	
1	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792	
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
67	6.7	13.4	20.1	26.8	33.5	40.2	46.9	53.6	60.3
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6

No. 675 L. 829.] [No. 719 L. 857.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
675	829304	9368	9432	9497	9561	9625	9690	9754	9818	9882	
6	9947										
		0011	0075	0139	0204	0268	0332	0396	0460	0525	
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	
1	9478	9541	9604	9667	9729	9792	9855	9918	9981		
										0043	
2	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671	
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	
8	850033	0095	0156	0217	0279	0340	0401	0462	0524	0585	
9	0046	0707	0769	0830	0891	0952	1014	1075	1136	1197	
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
63	6.3	12.6	18 9	25.2	31.5	37.8	44.1	50.4	56.7
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8
61	6.1	12.2	18.3	24.4	30.5	36.6	42.7	48.8	54.9
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0

No. 720 L. 857.] [No. 764 L. 883.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
720	857332	7393	7453	7513	7574	7634	7694	7755	7815	7875	
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978						
						0038	0098	0158	0218	0278	
5	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877	
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	
1	9818	9877	9935	9994							
					0053	0111	0170	0228	0287	0345	
2	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930	
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	
8	9669	9726	9784	9841	9898	9956					
							0013	0070	0127	0185	
9	880242	0299	0356	0413	0471	0528	0585	0642	0699	0756	
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4

No. 765 L. 883.] [No. 809 L. 908.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
765	883661	3718	3775	3832	3888	3945	4002	4059	4115	4172	
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974								
				0030	0086	0141	0197	0253	0309	0365	
7	890421	0477	0533	0589	0645	0700	0756	0812	0868	0924	
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	
4	9821	9875	9930	9985							
					0039	0094	0149	0203	0258	0312	
5	900367	0422	0476	0531	0586	0640	0695	0749	0804	0859	
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
55	5.5	11.0	16.5	22.0	27.5	33.0	38.5	44.0	49.5
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

No. 810 L. 908.] [No. 854 L. 931.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
810	908485	8539	8592	8646	8699	8753	8807	8860	8914	8967	
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	0037	
3	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571	
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	
1	9601	9653	9706	9758	9810	9862	9914	9967	0019	0071	
2	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	
1	9930	9981	0032	0083	0134	0185	0236	0287	0338	0389	51
2	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
53	5.3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47.7
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0

No. 855 L. 931.] [No. 899 L. 954.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
855	931966	2017	2068	2118	2169	2220	2271	2322	2372	2423	
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	
5	7016	7066	7116	7167	7217	7267	7317	7367	7418	7468	
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	
1	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	
5	6943	6992	7041	7090	7139	7189	7238	7287	7336	7385	
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	
8	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	
1	9878	9926	9975	0024	0073	0121	0170	0219	0267	0316	
2	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
49	4.9	9.8	14.7	19.6	24.5	29.4	34.3	39.2	44.1
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	
2	9995										
		0042	0090	0138	0185	0233	0280	0328	0376	0423	
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	
3	9882	9928	9975								
				0021	0068	0114	0161	0207	0254	0300	
4	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4

No. 945 L. 975.] [No. 989 L. 995.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
945	975432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
5	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	
7	9895	9939	9983								
				0028	0072	0117	0161	0206	0250	0294	
8	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	

Proportional Parts.

Diff.	1	2	3	4	5	6	7	8	9
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
45	4.5	9.0	13.5	18.0	22.5	27.0	31.5	36.0	40.5
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
43	4.3	8.6	12.9	17.2	21.5	25.8	30.1	34.4	38.7

No. 990 L. 995.] [No. 999 L. 999.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

Logarithms of Numbers from 1 to 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556302	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

LOGARITHMIC SINES, COSINES, TANGENTS AND COTANGENTS.

″	′	Sine.	$q-l$		Tang.	Cotang.	$q+l$	D 1″	Cosine.	′
			4.685				15.314			
0	0	Inf. neg.	575	575	Inf. neg.	Inf. pos.	425		ten	60
60	1	6.463726	575	575	6.463726	13.536274	425		ten	59
120	2	.764756	575	575	.764756	.235244	425		ten	58
180	3	6.940847	575	575	6.940847	13.059153	425		ten	57
240	4	7.065786	575	575	7.065786	12.934214	425		ten	56
300	5	.162696	575	575	.162696	.837304	425		ten	55
360	6	.241877	575	575	.241878	.758122	425	.02	9.999999	54
420	7	.308824	575	575	.308825	.691175	425	.00	.999999	53
480	8	.366816	574	576	.366817	.633183	424	.00	.999999	52
540	9	.417968	574	576	.417970	.582030	424	.00	.999999	51
600	10	.463726	574	576	.463727	.536273	424	.02	.999998	50
660	11	7.505118	574	576	7.505120	12.494880	424	.00	9.999998	49
720	12	.542906	574	577	.542909	.457091	423	.02	.999997	48
780	13	.577668	574	577	.577672	.422328	423	.00	.999997	47
840	14	.609853	574	577	.609857	.390143	423	.02	.999996	46
900	15	.639816	573	578	.639820	.360180	422	.00	.999996	45
960	16	.667845	573	578	.667849	.332151	422	.02	.999995	44
1020	17	.694173	573	578	.694179	.305821	422	.00	.999995	43
1080	18	.718997	573	579	.719003	.280997	421	.02	.999994	42
1140	19	.742478	573	579	.742484	.257516	421	.02	.999993	41
1200	20	.764754	572	580	.764761	.235239	420	.00	.999993	40
1260	21	7.785943	572	580	7.785951	12.214049	420	.02	9.999992	39
1320	22	.806146	572	581	.806155	.193845	419	.02	.999991	38
1380	23	.825451	572	581	.825460	.174540	419	.02	.999990	37
1440	24	.843934	571	582	.843944	.156056	418	.02	.999989	36
1500	25	.861662	571	583	.861674	.138326	417	.00	.999989	35
1560	26	.878695	571	583	.878708	.121292	417	.02	.999988	34
1620	27	.895085	570	584	.895099	.104901	416	.02	.999987	33
1680	28	.910879	570	584	.910894	.089106	416	.02	.999986	32
1740	29	.926119	570	585	.926134	.073866	415	.02	.999985	31
1800	30	.940842	569	586	.940858	.059142	414	.03	.999983	30
1860	31	7.955082	569	587	7.955100	12.044900	413	.02	9.999982	29
1920	32	.968870	569	587	.968889	.031111	413	.02	.999981	28
1980	33	.982233	568	588	.982253	.017747	412	.02	.999980	27
2040	34	7.995198	568	589	7.995219	12.004781	411	.02	.999979	26
2100	35	8.007787	567	590	8.007809	11.992191	410	.03	.999977	25
2160	36	.020021	567	591	.020044	.979956	409	.02	.999976	24
2220	37	.031919	566	592	.031945	.968055	408	.02	.999975	23
2280	38	.043501	566	593	.043527	.956473	407	.03	.999973	22
2340	39	.054781	566	593	.054809	.945191	407	.02	.999972	21
2400	40	.065776	565	594	.065806	.934194	406	.02	.999971	20
2460	41	8.076500	565	595	8.076531	11.923469	405	.03	9.999969	19
2520	42	.086965	564	596	.086997	.913003	404	.02	.999968	18
2580	43	.097183	564	598	.097217	.902783	402	.03	.999966	17
2640	44	.107167	563	599	.107203	.892797	401	.03	.999964	16
2700	45	.116926	562	600	.116963	.883037	400	.02	.999963	15
2760	46	.126471	562	601	.126510	.873490	399	.03	.999961	14
2820	47	.135810	561	602	.135851	.864149	398	.03	.999959	13
2880	48	.144953	561	603	.144996	.855004	397	.02	.999958	12
2940	49	.153907	560	604	.153952	.846048	396	.03	.999956	11
3000	50	.162681	560	605	.162727	.837273	395	.03	.999954	10
3060	51	8.171280	559	607	8.171328	11.828672	393	.03	9.999952	9
3120	52	.179713	558	608	.179763	.820237	392	.03	.999950	8
3180	53	.187985	558	609	.188036	.811964	391	.03	.999948	7
3240	54	.196102	557	611	.196156	.803844	389	.03	.999946	6
3300	55	.204070	556	612	.204126	.795874	388	.03	.999944	5
3360	56	.211895	556	613	.211953	.788047	387	.03	.999942	4
3420	57	.219581	555	615	.219641	.780359	385	.03	.999940	3
3480	58	.227134	554	616	.227195	.772805	384	.03	.999938	2
3540	59	.234557	554	618	.234621	.765379	382	.03	.999936	1
3600	60	8.241855	553	619	8.241921	11.758079	381	.03	9.999934	0
			4.685				15.314			
″	′	Cosine.	$q-l$		Cotang.	Tang.	$q+l$	D 1″	Sine.	′

″	′	Sine.	$q-l$		Tang.	Cotang.	$q+l$	D 1″	Cosine.	′
			4.685				15.314			
3600	0	8.241855	553	619	8.241921	11.758079	381	.03	9.999934	60
3660	1	.249033	552	620	.249102	.750898	380	.05	.999932	59
3720	2	.256094	551	622	.256165	.743835	378	.03	.999929	58
3780	3	.263042	551	623	.263115	.736885	377	.03	.999927	57
3840	4	.269881	550	625	.269956	.730044	375	.05	.999925	56
3900	5	.276614	549	627	.276691	.723309	373	.03	.999922	55
3960	6	.283243	548	628	.283323	.716677	372	.03	.999920	54
4020	7	.289773	547	630	.289856	.710144	370	.05	.999918	53
4080	8	.296207	546	632	.296292	.703708	368	.03	.999915	52
4140	9	.302546	546	633	.302634	.697366	367	.05	.999913	51
4200	10	.308794	545	635	.308884	.691116	365		.999910	50
4260	11	8.314954	544	637	8.315046	11.684954	363	.05	9.999907	49
4320	12	.321027	543	638	.321122	.678878	362	.03	.999905	48
4380	13	.327016	542	640	.327114	.672886	360	.05	.999902	47
4440	14	.332924	541	642	.333025	.666975	358	.05	.999899	46
4500	15	.338753	540	644	.338856	.661144	356	.03	.999897	45
4560	16	.344504	539	646	.344610	.655390	354	.05	.999894	44
4620	17	.350181	539	648	.350289	.649711	352	.05	.999891	43
4680	18	.355783	538	649	.355895	.644105	351	.05	.999888	42
4740	19	.361315	537	651	.361430	.638570	349	.05	.999885	41
4800	20	.366777	536	653	.366895	.633105	347		.999882	40
4860	21	8.372171	535	655	8.372292	11.627708	345	.05	9.999879	39
4920	22	.377499	534	657	.377622	.622378	343	.05	.999876	38
4980	23	.382762	533	659	.382889	.617111	341	.05	.999873	37
5040	24	.387962	532	661	.388092	.611908	339	.05	.999870	36
5100	25	.393101	531	663	.393234	.606766	337	.05	.999867	35
5160	26	.398179	530	666	.398315	.601685	334	.05	.999864	34
5220	27	.403199	529	668	.403338	.596662	332	.05	.999861	33
5280	28	.408161	527	670	.408304	.591696	330	.07	.999858	32
5340	29	.413068	526	672	.413213	.586787	328	.05	.999854	31
5400	30	.417919	525	674	.418068	.581932	326		.999851	30
5460	31	8.422717	524	676	8.422869	11.577131	324	.05	9.999848	29
5520	32	.427462	523	679	.427618	.572382	321	.07	.999844	28
5580	33	.432156	522	681	.432315	.567685	319	.05	.999841	27
5640	34	.436800	521	683	.436962	.563038	317	.05	.999838	26
5700	35	.441394	520	685	.441560	.558440	315	.07	.999834	25
5760	36	.445941	518	688	.446110	.553890	312	.05	.999831	24
5820	37	.450440	517	690	.450613	.549387	310	.07	.999827	23
5880	38	.454893	516	693	.455070	.544930	307	.05	.999824	22
5940	39	.459301	515	695	.459481	.540519	305	.07	.999820	21
6000	40	.463665	514	697	.463849	.536151	303	.07	.999816	20
6060	41	8.467985	512	700	8.468172	11.531828	300	.05	9.999813	19
6120	42	.472263	511	702	.472454	.527546	298	.07	.999809	18
6180	43	.476498	510	705	.476693	.523307	295	.07	.999805	17
6240	44	.480693	509	707	.480892	.519108	293	.07	.999801	16
6300	45	.484848	507	710	.485050	.514950	290	.05	.999797	15
6360	46	.488963	506	713	.489170	.510830	287	.07	.999794	14
6420	47	.493040	505	715	.493250	.506750	285	.07	.999790	13
6480	48	.497078	503	718	.497293	.502707	282	.07	.999786	12
6540	49	.501080	502	720	.501298	.498702	280	.07	.999782	11
6600	50	.505045	501	723	.505267	.494733	277	.07	.999778	10
6660	51	8.508974	499	726	8.509200	11.490800	274	.08	9.999774	9
6720	52	.512867	498	729	.513098	.486902	271	.07	.999769	8
6780	53	.516726	497	731	.516961	.483039	269	.07	.999765	7
6840	54	.520551	495	734	.520790	.479210	266	.07	.999761	6
6900	55	.524343	494	737	.524586	.475414	263	.07	.999757	5
6960	56	.528102	492	740	.528349	.471651	260	.08	.999753	4
7020	57	.531828	491	743	.532080	.467920	257	.07	.999748	3
7080	58	.535523	490	745	.535779	.464221	255	.07	.999744	2
7140	59	.539186	488	748	.539447	.460553	252	.08	.999740	1
7200	60	8.542819	487	751	8.543084	11.456916	249		9.999735	0
			4.685				15.314			
″	′	Cosine.	$q-l$		Cotang.	Tang.	$q+l$	D 1″	Sine.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8 542819	60.05	9.999735	.07	8.543084	60.12	11.456916	60
1	.546422	59.55	.999731	.08	.546691	59.62	.453309	59
2	.549995	59.07	.999726	.07	.550268	59.15	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.65	.446183	57
4	.557054	58.10	.999717	.07	.557336	58.20	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.72	.439172	55
6	.563999	57.20	.999708	.07	.564291	57.27	.435709	54
7	.567431	56.75	.999704	.08	.567727	56.83	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	.577566	55.43	.999689	.07	.577877	55.52	.422123	50
11	8.580892	55.02	9.999685	.08	8.581208	55.10	11.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.20	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.78	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.40	.999665	.08	.594283	53.48	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.62	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.85	.999645	.08	.606978	51.93	.393022	41
20	.609734	51.48	.999640	.08	.610094	51.58	.389906	40
21	8.612823	51.13	9.999635	.10	8.613189	51.22	11.386811	39
22	.615891	50.77	.999629	.08	.616262	50.85	.383738	38
23	.618937	50.42	.999624	.08	.619313	50.50	.380687	37
24	.621962	50.05	.999619	.08	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.10	.625352	49.80	.374648	35
26	.627948	49.38	.999608	.08	.628340	49.47	.371660	34
27	.630911	49.05	.999603	.10	.631308	49.13	.368692	33
28	.633854	48.70	.999597	.08	.634256	48.80	.365744	32
29	.636776	48.40	.999592	.10	.637184	48.48	.362816	31
30	.639680	48.05	.999586	.08	.640093	48.15	.359907	30
31	8.642563	47.75	9.999581	.10	8.642982	47.85	11.357018	29
32	.645428	47.43	.999575	.08	.645853	47.52	.354147	28
33	.648274	47.13	.999570	.10	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.10	.651537	46.92	.348463	26
35	.653911	46.52	.999558	.08	.654352	46.62	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.32	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.45	.334567	21
40	.667689	45.07	.999529	.08	.668160	45.17	.331840	20
41	8.670393	44.78	9.999524	.10	8.670870	44.88	11.329130	19
42	.673080	44.52	.999518	.10	.673563	44.60	.326437	18
43	.675751	44.23	.999512	.10	.676239	44.35	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	.681043	43.70	.999500	.12	.681544	43.80	.318456	15
46	.683665	43.45	.999493	.10	.684172	43.53	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.6914'8	42.67	.999475	.10	.691963	42.77	.308037	11
50	.693998	42.42	.999469	.10	.694529	42.53	.305471	10
51	8.696543	42.17	9.999463	.12	8.697081	42.27	11.302919	9
52	.699073	41.93	.999456	.10	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.12	.702139	41.78	.297861	7
54	.704090	41.45	.999443	.10	.704646	41.57	.295354	6
55	.706577	41.20	.999437	.10	.707140	41.30	.292860	5
56	.709049	40.97	.999431	.12	.709618	41.08	.290382	4
57	.711507	40.75	.999424	.10	.712083	40.85	.287917	3
58	.713952	40.52	.999418	.10	.714534	40.63	.285466	2
59	.716383	40.28	.999411	.12	.716972	40.40	.283028	1
60	8.718800		9.999404	.12	8.719396		11.280604	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.718800	40.07	9.999404	.10	8.719396	40.17	11.280604	60
1	.721204	39.85	.999398	.12	.721806	39.97	.278194	59
2	.723595	39.62	.999391	.12	.724204	39.73	.275796	58
3	.725972	39.42	.999384	.10	.726588	39.52	.273412	57
4	.728337	39.18	.999378	.12	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.12	.731317	39.10	.268683	55
6	.733027	38.78	.999364	.12	.733663	38.88	.266337	54
7	.735354	38.55	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.37	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.17	.999343	.12	.740626	38.27	.259374	51
10	.742259	37.95	.999336	.12	.742922	38.08	.257078	50
11	8.744536	37.77	9.999329	.12	8.745207	37.87	11.254793	49
12	.746802	37.55	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.48	.250260	47
14	.751297	37.18	.999308	.12	.751989	37.30	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.80	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.60	.999287	.13	.758668	36.73	.241332	43
18	.760151	36.43	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.23	.999272	.12	.763065	36.35	.236935	41
20	.764511	36.07	.999265	.13	.765246	36.18	.234754	40
21	8.766675	35.88	9.999257	.12	8.767417	36.02	11.232583	39
22	.768828	35.70	.999250	.13	.769578	35.82	.230422	38
23	.770970	35.52	.999242	.12	.771727	35.65	.228273	37
24	.773101	35.37	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.17	.999227	.12	.775995	35.32	.224005	35
26	.777333	35.02	.999220	.13	.778114	35.13	.221886	34
27	.779434	34.83	.999212	.12	.780222	34.97	.219778	33
28	.781524	34.68	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.50	.999197	.13	.784408	34.63	.215592	31
30	.785675	34.35	.999189	.13	.786486	34.47	.213514	30
31	8.787736	34.18	9.999181	.12	8.788554	34.32	11.211446	29
32	.789787	34.02	.999174	.13	.790613	34.15	.209387	28
33	.791828	33.85	.999166	.13	.792662	33.98	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.55	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.38	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.25	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.07	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	.805852	32.78	.999110	.13	.806742	32.92	.193258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.77	11.191283	19
42	.809777	32.48	.999094	.13	.810683	32.63	.189317	18
43	.811726	32.35	.999086	.15	.812641	32.47	.187359	17
44	.813667	32.20	.999077	.13	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.13	.816529	32.20	.183471	15
46	.817522	31.90	.999061	.13	.818461	32.05	.181539	14
47	.819436	31.78	.999053	.13	.820384	31.90	.179616	13
48	.821343	31.62	.999044	.15	.822298	31.78	.177702	12
49	.823240	31.50	.999036	.13	.824205	31.63	.175795	11
50	.825130	31.35	.999027	.15	.826103	31.48	.173897	10
51	8.827011	31.22	9.999019	.13	8.827992	31.37	11.172008	9
52	.828884	31.08	.999010	.15	.829874	31.23	.170126	8
53	.830749	30.97	.999002	.13	.831748	31.08	.168252	7
54	.832607	30.82	.998993	.15	.833613	30.97	.166387	6
55	.834456	30.68	.998984	.15	.835471	30.83	.164529	5
56	.836297	30.55	.998976	.13	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.58	.160837	3
58	.839956	30.30	.998958	.15	.840998	30.45	.159002	2
59	.841774	30.18	.998950	.13	.842825	30.32	.157175	1
60	8.843585		9.998941	.15	8.844644		11.155356	0
′	Cosine.	D 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.843585	30.03	9.998941	.15	8.844644	30.18	11.155356	60
1	.845387	29.93	.998932	.15	.846455	30.08	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.57	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.30	.998887	.15	.855403	29.47	.144597	54
7	.856049	29.20	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.95	.998860	.15	.860686	29.12	.139314	51
10	.861283	28.85	.998851	.15	.862433	29.00	.137567	50
11	8.863014	28.73	9.998841	.15	8.864173	28.88	11.135827	49
12	.864738	28.62	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.17	.867632	28.65	.132368	47
14	.868165	28.38	.998813	.15	.869351	28.55	.130649	46
15	.869868	28.28	.998804	.15	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.17	.872770	28.32	.127230	44
17	.873255	28.05	.998785	.15	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.17	.876162	28.12	.123838	42
19	.876615	27.83	.998766	.17	.877849	28.00	.122151	41
20	.878285	27.73	.998757	.17	.879529	27.88	.120471	40
21	8.879949	27.63	9.998747	.15	8.881202	27.78	11.118798	39
22	.881607	27.52	.998738	.17	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.17	.884530	27.58	.115470	37
24	.884903	27.32	.998718	.17	.886185	27.47	.113815	36
25	.886542	27.20	.998708	.17	.887833	27.38	.112167	35
26	.888174	27.12	.998699	.15	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.17	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.17	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	.894643	26.72	.998659	.17	.895984	26.87	.104016	30
31	8.896246	26.60	9.998649	.17	8.897596	26.78	11.102404	29
32	.897842	26.50	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.42	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.32	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.28	.094430	24
37	.905736	26.02	.998589	.18	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.85	.998568	.17	.910285	26.02	.089715	21
40	.910404	25.75	.998558	.17	.911846	25.92	.088154	20
41	8.911949	25.65	9.998548	.18	8.913401	25.83	11.086599	19
42	.913488	25.57	.998537	.17	.914951	25.73	.085049	18
43	.915022	25.47	.998527	.17	.916495	25.63	.083505	17
44	.916550	25.38	.998516	.18	.918034	25.57	.081966	16
45	.918073	25.30	.998506	.17	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.18	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.17	.922619	25.28	.077381	13
48	.922610	25.03	.998474	.17	.924136	25.22	.075864	12
49	.924112	24.95	.998464	.18	.925649	25.12	.074351	11
50	.925609	24.85	.998453	.18	.927156	25.03	.072844	10
51	8.927100	24.78	9.998442	.18	8.928658	24.95	11.071342	9
52	.928587	24.68	.998431	.17	.930155	24.87	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.30	.059506	1
60	8.940296		9.998344		8.941952		11.058048	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.940296	24.03	9.998344	.18	8.941952	24.20	11.058048	60
1	.941738	23.93	.998333	.18	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.18	.944852	24.05	.055148	58
3	.944606	23.80	.998311	.18	.946295	23.98	.053705	57
4	.946034	23.70	.998300	.18	.947734	23.90	.052266	56
5	.947456	23.63	.998289	.20	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.18	.950597	23.73	.049403	54
7	.950287	23.48	.998266	.18	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.20	.953441	23.58	.046559	52
9	.953100	23.32	.998243	.18	.954856	23.52	.045144	51
10	.954499	23.25	.998232	.20	.956267	23.45	.043733	50
11	8.955894	23.17	9.998220	.18	8.957674	23.35	11.042326	49
12	.957284	23.10	.998209	.20	.959075	23.30	.040925	48
13	.958670	23.03	.998197	.18	.960473	23.22	.039527	47
14	.960052	22.95	.998186	.20	.961866	23.15	.038134	46
15	.961429	22.87	.998174	.18	.963255	23.07	.036745	45
16	.962801	22.82	.998163	.20	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.20	.966019	22.92	.033981	43
18	.965534	22.65	.998139	.18	.967394	22.87	.032606	42
19	.966893	22.60	.998128	.20	.968766	22.78	.031234	41
20	.968249	22.52	.998116	.20	.970133	22.72	.029867	40
21	8.969600	22.45	9.998104	.20	8.971496	22.65	11.028504	39
22	.970947	22.37	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.32	.998080	.20	.974209	22.52	.025791	37
24	.973628	22.23	.998068	.20	.975560	22.43	.024440	36
25	.974962	22.18	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.25	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	.981573	21.83	.997996	.20	.983577	22.03	.016423	30
31	8.982883	21.77	9.997984	.20	8.984899	21.97	11.015101	29
32	.984189	21.72	.997972	.22	.986217	21.92	.013783	28
33	.985491	21.63	.997959	.20	.987532	21.83	.012468	27
34	.986789	21.57	.997947	.20	.988842	21.78	.011158	26
35	.988083	21.52	.997935	.22	.990149	21.70	.009851	25
36	.989374	21.43	.997922	.20	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.22	.992750	21.58	.007250	23
38	.991943	21.32	.997897	.20	.994045	21.53	.005935	22
39	.993222	21.25	.997885	.22	.995337	21.45	.004663	21
40	.994497	21.18	.997872	.20	.996624	21.40	.003376	20
41	8.995768	21.13	9.997860	.22	8.997908	21.33	11.002092	19
42	.997036	21.05	.997847	.20	8.999188	21.28	11.000812	18
43	.998299	21.02	.997835	.22	9.000465	21.22	10.999535	17
44	8.999560	20.93	.997822	.22	.001738	21.15	.998262	16
45	9.000816	20.88	.997809	.20	.003007	21.08	.996993	15
46	.002069	20.82	.997797	.22	.004272	21.03	.995728	14
47	.003318	20.75	.997784	.22	.005534	20.97	.944466	13
48	.004563	20.70	.997771	.22	.006792	20.92	.993208	12
49	.005805	20.65	.997758	.22	.008047	20.85	.991953	11
50	.007044	20.57	.997745	.22	.009298	20.80	.990702	10
51	9.008278	20.53	9.997732	.22	9.010546	20.73	10.989454	9
52	.009510	20.45	.997719	.22	.011790	20.68	.988210	8
53	.010737	20.42	.997706	.22	.013031	20.62	.986969	7
54	.011962	20.33	.997693	.22	.014268	20.57	.985732	6
55	.013182	20.30	.997680	.22	.015502	20.50	.984498	5
56	.014400	20.22	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.18	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.07	.997628	.23	.020403	20.28	.979597	1
60	9.019235		9.997614		9.021620		10.978380	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.88	.997588	.23	.024044	20.12	.975956	58
3	.022825	19.85	.997574	.22	.025251	20.07	.974749	57
4	.024016	19.78	.997561	.23	.026455	20.00	.973545	56
5	.025203	19.72	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.68	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.22	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.80	.968763	52
9	.029918	19.52	.997493	.22	.032425	19.73	.967575	51
10	.031089	19.47	.997480	.23	.033609	19.70	.966391	50
11	9.032257	19.40	9.997466	.23	9.034791	19.63	10.965209	49
12	.033421	19.35	.997452	.22	.035969	19.58	.964031	48
13	.034582	19.32	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.37	.959349	44
17	.039197	19.08	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	.042625	18.95	.997341	.23	.045284	19.17	.954716	40
21	9.043762	18.88	9.997327	.23	9.046434	19.13	10.953566	39
22	.044895	18.85	.997313	.23	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.23	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.23	.049869	18.98	.950131	36
25	.048279	18.68	.997271	.23	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.25	.052144	18.88	.947856	34
27	.050519	18.60	.997242	.23	.053277	18 83	.946723	33
28	.051635	18.57	.997228	.23	.054407	18.80	.945593	32
29	.052749	18.50	.997214	.25	.055535	18.73	.944465	31
30	.053859	18.45	.997199	.23	.056659	18.70	.943341	30
31	9.054966	18.42	9.997185	.25	9.057781	18.65	10.942219	29
32	.056071	18.35	.997170	.23	.058900	18.60	.941100	28
33	.057172	18.32	.997156	.25	.060016	18.57	.939984	27
34	.058271	18.27	.997141	.23	.061130	18.50	.938870	26
35	.059367	18.22	.997127	.25	.062240	18.47	.937760	25
36	.060460	18.18	.997112	.23	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.25	.064453	18.38	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.32	.934444	22
39	.063724	18.03	.997068	.25	.066655	18.28	.933345	21
40	.064806	17.98	.997053	.23	.067752	18.25	.932248	20
41	9.065885	17.95	9.997039	.25	9.068846	18.20	10.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.85	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.82	.996994	.25	.072113	18.07	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.73	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.67	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.88	.923568	12
49	.074424	17.60	.996919	.25	.077505	17.85	.922495	11
50	.075480	17.55	.996904	.25	.078576	17.80	.921424	10
51	9.076533	17.50	9.996889	.25	9.079644	17.77	10.920356	9
52	.077583	17.47	.996874	.27	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.27	.082833	17.63	.917167	6
55	.080719	17.33	.996828	.27	.083891	17.60	.916109	5
56	.081759	17.30	.996812	.25	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.25	.086000	17.50	.914000	3
58	.083832	17.20	.996782	.27	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.25	.088098	17.43	.911902	1
60	9.085894		9.996751		9.089144		10.910856	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.085894	17.13	9.996751	.27	9.089144	17.38	10.910856	60
1	.086922	17.08	.996735	.25	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.27	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.27	.092266	17.27	.907734	57
4	.089990	16.97	.996688	.25	.093302	17.23	.906698	56
5	.091008	16.93	.996673	.27	.094336	17.18	.905664	55
6	.092024	16.88	.996657	.27	.095367	17.13	.904633	54
7	.093037	16.83	.996641	.27	.096395	17.12	.903605	53
8	.094047	16.82	.996625	.25	.097422	17.07	.902578	52
9	.095056	16.77	.996610	.27	.098446	17.03	.901554	51
10	.096062	16.72	.996594	.27	.099468	16.98	.900532	50
11	9.097065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	.098066	16.65	.996562	.27	.101504	16.92	.898496	48
13	.099065	16.62	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.83	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.48	.996498	.27	.105550	16.77	.894450	44
17	.103037	16.47	.996482	.28	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.68	.892441	42
19	.105010	16.37	.996449	.27	.108560	16.65	.891440	41
20	.105992	16.35	.996433	.27	.109559	16.62	.890441	40
21	9.106973	16.30	9.996417	.28	9.110556	16.58	10.889444	39
22	.107951	16.27	.996400	.27	.111551	16.53	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.20	.996368	.28	.113533	16.47	.886467	36
25	.110873	16.15	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.28	.115507	16.40	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.35	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.33	.882528	32
29	.114737	16.02	.996285	.27	.118452	16.28	.881548	31
30	.115698	15.97	.996269	.28	.119429	16.25	.880571	30
31	9.116656	15.95	9.996252	.28	9.120404	16.22	10.879596	29
32	.117613	15.90	.996235	.27	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.12	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.75	.996168	.28	.125249	16.03	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.02	.873789	23
38	.123306	15.70	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.65	.996117	.28	.128130	15.95	.871870	21
40	.125187	15.63	.996100	.28	.129087	15.90	.870913	20
41	9.126125	15.58	9.996083	.28	9.130041	15.88	10.869959	19
42	.127060	15.55	.996066	.28	.130994	15.83	.869006	18
43	.127993	15.53	.996049	.28	.131944	15.82	.868056	17
44	.128925	15.48	.996032	.28	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.28	.133839	15.75	.866161	15
46	.130781	15.42	.995998	.30	.134784	15.70	.865216	14
47	.131706	15.40	.995980	.28	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.28	.136667	15.63	.863333	12
49	.133551	15.32	.995946	.30	.137605	15.62	.862395	11
50	.134470	15.28	.995928	.28	.138542	15.57	.861458	10
51	9.135387	15.27	9.995911	.28	9.139476	15.55	10.860524	9
52	.136303	15.22	.995894	.30	.140409	15.52	.859591	8
53	.137216	15.20	.995876	.28	.141340	15.48	.858660	7
54	.138128	15.15	.995859	.30	.142269	15.45	.857731	6
55	.139037	15.12	.995841	.30	.143196	15.42	.856804	5
56	.139944	15.10	.995823	.28	.144121	15.38	.855879	4
57	.140850	15.07	.995806	.30	.145044	15.37	.854956	3
58	.141754	15.02	.995788	.28	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.30	.146885	15.30	.853115	1
60	9.143555		9.995753		9.147803		10.852197	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1′.	Tang.	D. 1″.	Cotang.	′
0	9.143555	14.97	9.995753	.30	9.147803	15.25	10.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.88	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.83	.995681	.28	.151454	15.15	.848546	56
5	.148026	14.82	.995664	.30	.152363	15.10	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.73	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.32	.155077	15.02	.844923	52
9	.151569	14.70	.995591	.30	.155978	14.98	.844022	51
10	.152451	14.65	.995573	.30	.156877	14.97	.843123	50
11	9.153330	14.63	9.995555	.30	9.157775	14.93	10.842225	49
12	.154208	14.58	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.55	.995501	.32	.160457	14.83	.839543	46
15	.156830	14.50	.995482	.30	.161347	14.82	.838653	45
16	.157700	14.48	.995464	.30	.162236	14.78	.837764	44
17	.158569	14.43	.995446	.32	.163123	14.75	.836877	43
18	.159435	14.43	.995427	.30	.164008	14.73	.835992	42
19	.160301	14.38	.995409	.32	.164892	14.70	.835108	41
20	.161164	14.35	.995390	.30	.165774	14.67	.834226	40
21	9.162025	14.33	9.995372	.32	9.166654	14.63	10.833346	39
22	.162885	14.30	.995353	.32	.167532	14.62	.832468	38
23	.163743	14.28	.995334	.30	.168409	14.58	.831591	37
24	.164600	14.23	.995316	.32	.169284	14.55	.830716	36
25	.165454	14.22	.995297	.32	.170157	14.53	.829843	35
26	.166307	14.20	.995278	.30	.171029	14.50	.828971	34
27	.167159	14.15	.995260	.32	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.45	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	.169702	14.08	.995203	.32	.174499	14.38	.825501	30
31	9.170547	14.03	9.995184	.32	9.175362	14.37	10.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	14.00	.995146	.32	.177084	14.30	.822916	27
34	.173070	13.97	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.93	.995108	.32	.178799	14.27	.821201	25
36	.174744	13.90	.995089	.32	.179655	14.22	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.18	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.13	.817789	21
40	.178072	13.80	.995013	.33	.183059	14.13	.816941	20
41	9.178900	13.77	9.994993	.32	9.183907	14.08	10.816093	19
42	.179726	13.75	.994974	.32	.184752	14.08	.815248	18
43	.180551	13.72	.994955	.33	.185597	14.03	.814403	17
44	.181374	13.70	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.33	.187280	14.00	.812720	15
46	.183016	13.63	.994896	.32	.188120	13.97	.811880	14
47	.183834	13.62	.994877	.33	.188958	13.93	.811042	13
48	.184651	13.58	.994857	.32	.189794	13.92	.810206	12
49	.185466	13.57	.994838	.33	.190629	13.88	.809371	11
50	.186280	13.53	.994818	.33	.191462	13.87	.808538	10
51	9.187092	13.52	9.994798	.32	9.192294	13.83	10.807706	9
52	.187903	13.48	.994779	.33	.193124	13.82	.806876	8
53	.188712	13.45	.994759	.33	.193953	13.78	.806047	7
54	.189519	13.43	.994739	.32	.194780	13.77	.805220	6
55	.190325	13.42	.994720	.33	.195606	13.73	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.72	.803570	4
57	.191933	13.35	.994680	.33	.197253	13.68	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.67	.801926	2
59	.193534	13.30	.994640	.33	.198894	13.65	.801106	1
60	9.194332		9.994620		9.199713		10.800287	0
′	Cosine.	D. 1″.	Sine.	D. 1′.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.194332	13.28	9.994620	.33	9.199713	13.60	10.800287	60
1	.195129	13.27	.994600	.33	.200529	13.60	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.20	.994560	.33	.202159	13.53	.797841	57
4	.197511	13.18	.994540	.33	.202971	13.52	.797029	56
5	.198302	13.15	.994519	.33	.203782	13.50	.796218	55
6	.199091	13.13	.994499	.33	.204592	13.47	.795408	54
7	.199879	13.12	.994479	.33	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.33	.206207	13.43	.793793	52
9	.201451	13.05	.994438	.33	.207013	13.40	.792987	51
10	.202234	13.05	.994418	.33	.207817	13.37	.792183	50
11	9.203017	13.00	9.994398	.33	9.208619	13.35	10.791381	49
12	.203797	13.00	.994377	.33	.209420	13.33	.790580	48
13	.204577	12.95	.994357	.33	.210220	13.30	.789780	47
14	.205354	12.95	.994336	.33	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.33	.211815	13.27	.788185	45
16	.206906	12.88	.994295	.33	.212611	13.23	.787389	44
17	.207679	12.88	.994274	.33	.213405	13.22	.786595	43
18	.208452	12.83	.994254	.33	.214198	13.18	.785802	42
19	.209222	12.83	.994233	.33	.214989	13.18	.785011	41
20	.209992	12.80	.994212	.33	.215780	13.13	.784220	40
21	9.210760	12.77	9.994191	.33	9.216568	13.13	10.783432	39
22	.211526	12.75	.994171	.33	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.33	.218142	13.07	.781858	37
24	.213055	12.72	.994129	.33	.218926	13.07	.781074	36
25	.213818	12.68	.994108	.33	.219710	13.03	.780290	35
26	.214579	12.65	.994087	.33	.220492	13.00	.779508	34
27	.215338	12.65	.994066	.33	.221272	13.00	.778728	33
28	.216097	12.62	.994045	.33	.222052	12.97	.777948	32
29	.216854	12.58	.994024	.33	.222830	12.95	.777170	31
30	.217609	12.57	.994003	.33	.223607	12.92	.776393	30
31	9.218363	12.55	9.993982	.37	9.224382	12.90	10.775618	29
32	.219116	12.53	.993960	.33	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.33	.225929	12.85	.774071	27
34	.220618	12.48	.993918	.33	.226700	12.85	.773300	26
35	.221367	12.47	.993897	.33	.227471	12.80	.772529	25
36	.222115	12.43	.993875	.37	.228239	12.80	.771761	24
37	.222861	12.42	.993854	.33	.229007	12.77	.770993	23
38	.223606	12.38	.993832	.37	.229773	12.77	.770227	22
39	.224349	12.38	.993811	.33	.230539	12.72	.769461	21
40	.225092	12.35	.993789	.37	.231302	12.72	.768698	20
41	9.225833	12.33	9.993768	.37	9.232065	12.68	10.767935	19
42	.226573	12.30	.993746	.33	.232826	12.67	.767174	18
43	.227311	12.28	.993725	.37	.233586	12.65	.766414	17
44	.228048	12.27	.993703	.37	.234345	12.63	.765655	16
45	.228784	12.23	.993681	.33	.235103	12.60	.764897	15
46	.229518	12.23	.993660	.37	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.37	.236614	12.57	.763386	13
48	.230984	12.18	.993616	.37	.237368	12.53	.762632	12
49	.231715	12.15	.993594	.37	.238120	12.53	.761880	11
50	.232444	12.13	.993572	.37	.238872	12.50	.761128	10
51	9.233172	12.12	9.993550	.37	9.239622	12.48	10.760378	9
52	.233899	12.10	.993528	.37	.240371	12.45	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.45	.758882	7
54	.235349	12.07	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.00	.993440	.37	.243354	12.38	.756646	4
57	.237515	12.00	.993418	.37	.244097	12.37	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.33	.755161	2
59	.238953	11.95	.993374	.38	.245579	12.33	.754421	1
60	9.239670		9.993351		9.246319		10.753681	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60
1	.240388	11.92	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.88	.993307	.38	.247794	12.27	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.23	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.23	.750736	56
5	.243237	11.83	.993240	.38	.249998	12.20	.750002	55
6	.243947	11.82	.993217	.37	.250730	12.18	.749270	54
7	.244656	11.78	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.77	.993149	.38	.252920	12.13	.747080	51
10	.246775	11.72	.993127	.38	.253648	12.10	.746352	50
11	9.247478	11.72	9.993104	.38	9.254374	12.10	10.745626	49
12	.248181	11.70	.993081	.37	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.02	.742731	45
16	.250980	11.62	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.60	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.57	.992944	.38	.259429	11.95	.740571	42
19	.253067	11.57	.992921	.38	.260146	11.95	.739854	41
20	.253761	11.53	.992898	.38	.260863	11.92	.739137	40
21	9.254453	11.52	9.992875	.38	9.261578	11.90	10.738422	39
22	.255144	11.50	.992852	.38	.262292	11.88	.737708	38
23	.255834	11.48	.992829	.38	.263005	11.87	.736995	37
24	.256523	11.47	.992806	.38	.263717	11.85	.736283	36
25	.257211	11.45	.992783	.38	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.40	.265138	11.82	.734862	34
27	.258583	11.42	.992736	.38	.265847	11.80	.734153	33
28	.259268	11.38	.992713	.38	.266555	11.77	.733445	32
29	.259951	11.37	.992690	.38	.267261	11.77	.732739	31
30	.260633	11.35	.992666	.40	.267967	11.73	.732033	30
31	9.261314	11.33	9.992643	.38	9.268671	11.73	10.731329	29
32	.261994	11.32	.992619	.40	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.38	.270077	11.70	.729923	27
34	.263351	11.27	.992572	.40	.270779	11.67	.729221	26
35	.264027	11.27	.992549	.38	.271479	11.65	.728521	25
36	.264703	11.23	.992525	.40	.272178	11.63	.727822	24
37	.265377	11.23	.992501	.40	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.38	.273573	11.60	.726427	22
39	.266723	11.20	.992454	.40	.274269	11.58	.725731	21
40	.267395	11.17	.992430	.40	.274964	11.57	.725036	20
41	9.268065	11.15	9.992406	.40	9.275658	11.55	10.724342	19
42	.268734	11.13	.992382	.38	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.52	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.07	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.03	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.42	.280488	11.43	.719512	12
49	.273388	11.02	.992214	.40	.281174	11.40	.718826	11
50	.274049	10.98	.992190	.40	.281858	11.40	.718142	10
51	9.274708	10.98	9.992166	.40	9.282542	11.38	10.717458	9
52	.275367	10.97	.992142	.40	.283225	11.37	.716775	8
53	.276025	10.93	.992118	.42	.283907	11.35	.716093	7
54	.276681	10.93	.992093	.40	.284588	11.33	.715412	6
55	.277337	10.90	.992069	.42	.285268	11.32	.714732	5
56	.277991	10.90	.992044	.40	.285947	11.28	.714053	4
57	.278645	10.87	.992020	.40	.286624	11.28	.713376	3
58	.279297	10.85	.991996	.42	.287301	11.27	.712699	2
59	.279948	10.85	.991971	.40	.287977	11.25	.712023	1
60	9.280599		9.991947		9.288652		10.711348	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.280599	10.82	9.991947	.42	9.288652	11.23	10.711348	60
1	.281248	10.82	.991922	.42	.289326	11.22	.710674	59
2	.281897	10.78	.991897	.40	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.42	.290671	11.18	.709329	57
4	.283190	10.77	.991848	.42	.291342	11.18	.708658	56
5	.283836	10.73	.991823	.40	.292013	11.15	.707987	55
6	.284480	10.73	.991799	.42	.292682	11.13	.707318	54
7	.285124	10.70	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.70	.991749	.42	.294017	11.12	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.08	.705316	51
10	.287048	10.67	.991699	.42	.295349	11.07	.704651	50
11	9.287688	10.63	9.991674	.42	9.296013	11.07	10.703987	49
12	.288326	10.63	.991649	.42	.296677	11.03	.703323	48
13	.288964	10.60	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.60	.991599	.42	.298001	11.02	.701999	46
15	.290236	10.57	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.57	.991549	.42	.299322	10.97	.700678	44
17	.291504	10.55	.991524	.43	.299980	10.97	.700020	43
18	.292137	10.52	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.52	.991473	.42	.301295	10.93	.698705	41
20	.293399	10.50	.991448	.43	.301951	10.93	.698049	40
21	9.294029	10.48	9.991422	.42	9.302607	10.90	10.697393	39
22	.294658	10.47	.991397	.42	.303261	10.88	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.88	.696086	37
24	.295913	10.43	.991346	.42	.304567	10.85	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.85	.694782	35
26	.297164	10.40	.991295	.42	.305869	10.83	.694131	34
27	.297788	10.40	.991270	.43	.306519	10.82	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.35	.991218	.42	.307816	10.78	.692184	31
30	.299655	10.35	.991193	.43	.308463	10.77	.691537	30
31	9.300276	10.32	9.991167	.43	9.309109	10.75	10.690891	29
32	.300895	10.32	.991141	.43	.309754	10.75	.690246	28
33	.301514	10.30	.991115	.42	.310399	10.72	.689601	27
34	.302132	10.27	.991090	.43	.311042	10.72	.688958	26
35	.302748	10.27	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.23	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.63	.685753	21
40	.305819	10.18	.990934	.43	.314885	10.63	.685115	20
41	9.306430	10.18	9.990908	.43	9.315523	10.60	10.684477	19
42	.307041	10.15	.990882	.45	.316159	10.60	.683841	18
43	.307650	10.15	.990855	.43	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.43	.317430	10.57	.682570	16
45	.308867	10.12	.990803	.43	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.45	.318697	10.55	.681303	14
47	.310080	10.08	.990750	.43	.319330	10.52	.680670	13
48	.310685	10.07	.990724	.45	.319961	10.52	.680039	12
49	.311289	10.07	.990697	.43	.320592	10.50	.679408	11
50	.311893	10.03	.990671	.43	.321222	10.48	.678778	10
51	9.312495	10.03	9.990645	.45	9.321851	10.47	10.678149	9
52	.313097	10.02	.990618	.45	.322479	10.45	.677521	8
53	.313698	9.98	.990591	.43	.323106	10.45	.676894	7
54	.314297	10.00	.990565	.45	.323733	10.42	.676267	6
55	.314897	9.97	.990538	.45	.324358	10.42	.675642	5
56	.315495	9.95	.990511	.43	.324983	10.40	.675017	4
57	.316092	9.95	.990485	.45	.325607	10.40	.674393	3
58	.316689	9.92	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.92	.990431	.45	.326853	10.37	.673147	1
60	9.317879		9.990404		9.327475		10.672525	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.317879	9.90	9.990404	.43	9.327475	10.33	10.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.85	.990324	.45	.329334	10.32	.670666	57
4	.320249	9.85	.990297	.45	.329953	10.28	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.47	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.78	.990188	.45	.332418	10.25	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.22	.666967	51
10	.323780	9.77	.990134	.45	.333646	10.22	.666354	50
11	9.324366	9.73	9.990107	.47	9.334259	10.20	10.665741	49
12	.324950	9.73	.990079	.45	.334871	10.18	.665129	48
13	.325534	9.72	.990052	.45	.335482	10.18	.664518	47
14	.326117	9.72	.990025	.47	.336093	10.15	.663907	46
15	.326700	9.68	.989997	.45	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.47	.337311	10.13	.662689	44
17	.327862	9.67	.989942	.45	.337919	10.13	.662081	43
18	.328442	9.65	.989915	.47	.338527	10.10	.661473	42
19	.329021	9.63	.989887	.45	.339133	10.10	.660867	41
20	.329599	9.62	.989860	.47	.339739	10.08	.660261	40
21	9.330176	9.62	9.989832	.47	9.340344	10.07	10.659656	39
22	.330753	9.60	.989804	.45	.340948	10.07	.659052	38
23	.331329	9.57	.989777	.47	.341552	10.05	.658448	37
24	.331903	9.58	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.55	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.55	.989693	.47	.343358	10.00	.656642	34
27	.333624	9.52	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.53	.989637	.45	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	.335337	9.48	.989582	.48	.345755	9.97	.654245	30
31	9.335906	9.48	9.989553	.47	9.346353	9.93	10.653647	29
32	.336475	9.47	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.93	.652455	27
34	.337610	9.43	.989469	.47	.348141	9.90	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.42	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.48	.349922	9.87	.650078	23
38	.339871	9.38	.989356	.47	.350514	9.87	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	.340996	9.37	.989300	.48	.351697	9.83	.648303	20
41	9.341558	9.35	9.989271	.47	9.352287	9.82	10.647713	19
42	.342119	9.33	.989243	.48	.352876	9.82	.647124	18
43	.342679	9.33	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.30	.989186	.48	.354053	9.78	.645947	16
45	.343797	9.30	.989157	.48	.354640	9.78	.645360	15
46	.344355	9.28	.989128	.47	.355227	9.77	.644773	14
47	.344912	9.28	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.25	.989071	.48	.356398	9.73	.643602	12
49	.346024	9.25	.989042	.47	.356982	9.73	.643018	11
50	.346579	9.25	.989014	.48	.357566	9.72	.642434	10
51	9.347134	9.22	9.988985	.48	9.358149	9.70	10.641851	9
52	.347687	9.22	.988956	.48	.358731	9.70	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.67	.640687	7
54	.348792	9.18	.988898	.48	.359893	9.68	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.65	.639526	5
56	.349893	9.17	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.13	.988782	.48	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.48	.362787	9.62	.637213	1
60	9.352088		9.988724		9.363364		10.636636	0
′	Cosine.	D. 1″.	Sine,	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.352088		9.988724		9.363364		10.636636	60
1	.352635	9.12	.988695	.48	.363940	9.60	.636060	59
2	.353181	9.10	.988666	.48	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.50	.365090	9.58	.634910	57
4	.354271	9.08	.988607	.48	.365664	9.57	.634336	56
5	.354815	9.07	.988578	.48	.366237	9.55	.633763	55
6	.355358	9.05	.988548	.50	.366810	9.55	.633190	54
7	.355901	9.05	.988519	.48	.367382	9.53	.632618	53
8	.356443	9.03	.988489	.50	.367953	9.52	.632047	52
9	.356984	9.02	.988460	.48	.368524	9.52	.631476	51
10	.357524	9.00	.988430	.50	.369094	9.50	.630906	50
11	9.358064	9.00	9.988401	.48	9.369663	9.48	10.630337	49
12	.358603	8.98	.988371	.50	.370232	9.48	.629768	48
13	.359141	8.97	.988342	.48	.370799	9.45	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.47	.628633	46
15	.360215	8.95	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.95	.988252	.50	.372499	9.43	.627501	44
17	.361287	8.92	.988223	.48	.373064	9.42	.626936	43
18	.361822	8.92	.988193	.50	.373629	9.42	.626371	42
19	.362356	8.90	.988163	.50	.374193	9.40	.625807	41
20	.362889	8.88	.988133	.50	.374756	9.38	.625244	40
21	9.363422	8.88	9.988103	.50	9.375319	9.38	10.624681	39
22	.363954	8.87	.988073	.50	.375881	9.37	.624119	38
23	.364485	8.85	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.85	.988013	.50	.377003	9.35	.622997	36
25	.365546	8.83	.987983	.50	.377563	9.33	.622437	35
26	.366075	8.82	.987953	.52	.378122	9.32	.621878	34
27	.366604	8.82	.987922	.50	.378681	9.32	.621319	33
28	.367131	8.78	.987892	.50	.379239	9.30	.620761	32
29	.367659	8.80	.987862	.50	.379797	9.30	.620203	31
30	.368185	8.77	.987832	.52	.380354	9.28	.619646	30
31	9.368711	8.77	9.987801	.50	9.380910	9.27	10.619090	29
32	.369236	8.75	.987771	.52	.381466	9.23	.618534	28
33	.369761	8.75	.987740	.50	.382020	9.25	.617980	27
34	.370285	8.72	.987710	.52	.382575	9.23	.617425	26
35	.370808	8.72	.987679	.50	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.52	.383682	9.20	.616318	24
37	.371852	8.70	.987618	.50	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.52	.384786	9.18	.615214	22
39	.372894	8.68	.987557	.52	.385337	9.18	.614663	21
40	.373414	8.67	.987526	.50	.385888	9.17	.614112	20
41	9.373933	8.65	9.987496	.52	9.386438	9.15	10.613562	19
42	.374452	8.65	.987465	.52	.386987	9.15	.613013	18
43	.374970	8.63	.987434	.52	.387536	9.13	.612464	17
44	.375487	8.62	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.12	.611369	15
46	.376519	8.60	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.60	.987310	.52	.389724	9.10	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.57	.987248	.52	.390815	9.08	.609185	11
50	.378577	8.57	.987217	.52	.391360	9.05	.608640	10
51	9.379089	8.53	9.987186	.52	9.391903	9.07	10.608097	9
52	.379601	8.53	.987155	.52	.392447	9.03	.607553	8
53	.380113	8.52	.987124	.53	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.03	.606469	6
55	.381134	8.48	.987061	.52	.394073	9.02	.605927	5
56	.381643	8.48	.987030	.53	.394614	9.00	.605386	4
57	.382152	8.48	.986998	.52	.395154	9.00	.604846	3
58	.382661	8.45	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.53	.396233	8.97	.603767	1
60	9.383675		9.986904		9.396771		10.603229	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.383675	8.45	9.986904	.52	9.396771	8.97	10.603229	60
1	.384182	8.42	.986873	.53	.397309	8.95	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.42	.986809	.53	.398383	8.93	.601617	57
4	.385697	8.40	.986778	.52	.398919	8.93	.601081	56
5	.386201	8.38	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.52	.399990	8.90	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.35	.986651	.53	.401058	8.88	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	.388711	8.33	.986587	.53	.402124	8.87	.597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.85	10.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.30	.986491	.53	.403718	8.85	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.82	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.83	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.80	.594692	44
17	.392199	8.27	.986363	.53	.405836	8.80	.594164	43
18	.392695	8.27	.986331	.53	.406364	8.80	.593636	42
19	.393191	8.23	.986299	.55	.406892	8.78	.593108	41
20	.393685	8.23	.986266	.53	.407419	8.77	.592581	40
21	9.394179	8.23	9.986234	.53	9.407945	8.77	10.592055	39
22	.394673	8.22	.986202	.55	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.53	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.55	.409521	8.73	.590479	36
25	.396150	8.18	.986104	.53	.410045	8.73	.589955	35
26	.396641	8.18	.986072	.55	.410569	8.72	.589431	34
27	.397132	8.15	.986039	.53	.411092	8.72	.588908	33
28	.397621	8.17	.986007	.55	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.53	.412137	8.68	.587863	31
30	.398600	8.13	.985942	.55	.412658	8.68	.587342	30
31	9.399088	8.12	9.985909	.55	9.413179	8.67	10.586821	29
32	.399575	8.12	.985876	.55	.413699	8.67	.586301	28
33	.400062	8.12	.985843	.53	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.08	.985778	.55	.415257	8.63	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.05	.985679	.55	.416810	8.60	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	.403455	8.05	.985613	.55	.417842	8.60	.582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.02	.985514	.57	.419387	8.57	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.57	.580099	16
45	.405862	7.98	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.55	.420927	8.55	.579073	14
47	.406820	7.98	.985381	.57	.421440	8.53	.578560	13
48	.407299	7.97	.985347	.55	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.57	.422463	8.52	.577537	11
50	.408254	7.95	.985280	.55	.422974	8.50	.577026	10
51	9.408731	7.93	9.985247	.57	9.423484	8.48	10.576516	9
52	.409207	7.93	.985213	.55	.423993	8.50	.576007	8
53	.409682	7.92	.985180	.57	.424503	8.47	.575497	7
54	.410157	7.92	.985146	.55	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.57	.425519	8.47	.574481	5
56	.411106	7.88	.985079	.57	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.57	.426534	8.45	.573466	3
58	.412052	7.87	.985011	.55	.427041	8.43	.572959	2
59	.412524	7.87	.984978	.57	.427547	8.42	.572453	1
60	9.412996		9.984944		9.428052		10.571948	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.412996	7.85	9.984944	.57	9.428052	8.43	10.571948	60
1	.413467	7.85	.984910	.57	.428558	8.40	.571442	59
2	.413938	7.83	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.40	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.80	.984774	.57	.430573	8.37	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.80	.984706	.57	.431577	8.37	.568423	53
8	.416751	7.77	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.78	.984638	.58	.432580	8.33	.567420	51
10	.417684	7.77	.984603	.57	.433080	8.33	.566920	50
11	9.418150	7.75	9.984569	.57	9.433580	8.33	10.566420	49
12	.418615	7.73	.984535	.58	.434080	8.32	.565920	48
13	.419079	7.75	.984500	.57	.434579	8.32	.565421	47
14	.419544	7.72	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.28	.564424	45
16	.420470	7.72	.984397	.57	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.70	.984328	.57	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.27	.562437	41
20	.422318	7.67	.984259	.58	.438059	8.25	.561941	40
21	9.422778	7.67	9.984224	.57	9.438554	8.23	10.561446	39
22	.423238	7.65	.984190	.58	.439048	8.25	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.22	.560457	37
24	.424156	7.65	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.22	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.62	.984015	.57	.441514	8.20	.558486	33
28	.425987	7.60	.983981	.58	.442006	8.18	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	.426899	7.58	.983911	.60	.442988	8.18	.557012	30
31	9.427354	7.58	9.983875	.58	9.443479	8.15	10.556521	29
32	.427809	7.57	.983840	.58	.443968	8.17	.556032	28
33	.428263	7.57	.983805	.58	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.58	.444947	8.13	.555053	26
35	.429170	7.55	.983735	.58	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.60	.445923	8.13	.554077	24
37	.430075	7.53	.983664	.58	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.58	.446898	8.10	.553102	22
39	.430978	7.52	.983594	.60	.447384	8.10	.552616	21
40	.431429	7.50	.983558	.58	.447870	8.10	.552130	20
41	9.431879	7.50	9.983523	.60	9.448356	8.08	10.551644	19
42	.432329	7.48	.983487	.58	.448841	8.08	.551159	18
43	.432778	7.47	.983452	.60	.449326	8.07	.550674	17
44	.433226	7.48	.983416	.58	.449810	8.07	.550190	16
45	.433675	7.45	.983381	.60	.450294	8.05	.549706	15
46	.434122	7.45	.983345	.60	.450777	8.05	.549223	14
47	.434569	7.45	.983309	.60	.451260	8.05	.548740	13
48	.435016	7.45	.983273	.60	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.58	.452225	8.02	.547775	11
50	.435908	7.43	.983202	.60	.452706	8.02	.547294	10
51	9.436353	7.42	9.983166	.60	9.453187	8.02	10.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.38	.983058	.60	.454628	7.98	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.37	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.35	.982914	.60	.456542	7.95	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60	9.440338		9.982842		9.457496		10.542504	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.440338	7.33	9.982842	.62	9.457496	7.95	10.542504	60
1	.440778	7.33	.982805	.60	.457973	7.93	.542027	59
2	.441218	7.33	.982769	.60	.458449	7.93	.541551	58
3	.441658	7.30	.982733	.62	.458925	7.92	.541075	57
4	.442096	7.32	.982696	.60	.459400	7.92	.540600	56
5	.442535	7.30	.982660	.60	.459875	7.90	.540125	55
6	.442973	7.28	.982624	.62	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.60	.460823	7.90	.539177	53
8	.443847	7.28	.982551	.62	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.62	.461770	7.87	.538230	51
10	.444720	7.25	.982477	.60	.462242	7.88	.537758	50
11	9.445155	7.25	9.982441	.62	9.462715	7.85	10.537285	49
12	.445590	7.25	.982404	.62	.463186	7.87	.536814	48
13	.446025	7.23	.982367	.60	.463658	7.83	.536342	47
14	.446459	7.23	.982331	.62	.464128	7.85	.535872	46
15	.446893	7.22	.982294	.62	.464599	7.83	.535401	45
16	.447326	7.22	.982257	.62	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.82	.533992	42
19	.448623	7.18	.982146	.62	.466477	7.80	.533523	41
20	.449054	7.18	.982109	.62	.466945	7.80	.533055	40
21	9.449485	7.17	9.982072	.62	9.467413	7.78	10.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.17	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.13	.981924	.63	.469280	7.77	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.75	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.63	.470676	7.75	.529324	32
29	.452915	7.12	.981774	.62	.471141	7.73	.528859	31
30	.453342	7.10	.981737	.62	.471605	7.73	.528395	30
31	9.453768	7.10	9.981700	.63	9.472069	7.72	10.527931	29
32	.454194	7.08	.981662	.62	.472532	7.72	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.70	.527005	27
34	.455044	7.08	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.62	.473919	7.70	.526081	25
36	.455893	7.05	.981512	.63	.474381	7.68	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.05	.981436	.62	.475303	7.67	.524697	22
39	.457162	7.03	.981399	.63	.475763	7.67	.524237	21
40	.457584	7.03	.981361	.63	.476223	7.67	.523777	20
41	9.458006	7.02	9.981323	.63	9.476683	7.65	10.523317	19
42	.458427	7.02	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.00	.981247	.63	.477601	7.63	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	7.00	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.63	.479432	7.62	.520568	13
48	.460946	6.97	.981057	.63	.479889	7.60	.520111	12
49	.461364	6.97	.981019	.63	.480345	7.60	.519655	11
50	.461782	6.95	.980981	.65	.480801	7.60	.519199	10
51	9.462199	6.95	9.980942	.63	9.481257	7.58	10.518743	9
52	.462616	6.93	.980904	.63	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.65	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.63	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.65	.483075	7.57	.516925	5
56	.464279	6.92	.980750	.63	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.65	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.63	.484435	7.53	.515565	2
59	.465522	6.88	.980635	.65	.484887	7.53	.515113	1
60	9.465935		9.980596		9.485339		10.514661	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.465935	6.88	9.980596	.63	9.485339	7.53	10.514661	60
1	.466348	6.88	.980558	.65	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.52	.513758	58
3	.467173	6.87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.83	.980364	.65	.488043	7.48	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	.470046	6.82	.980208	.65	.489838	7.47	.510162	50
11	9.470455	6.80	9.980169	.65	9.490286	7.45	10.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.80	.980091	.65	.491180	7.45	.508820	47
14	.471679	6.78	.980052	.67	.491627	7.43	.508373	46
15	.472086	6.77	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.77	.979934	.65	.492965	7.42	.507035	43
18	.473304	6.77	.979895	.67	.493410	7.40	.506590	42
19	.473710	6.75	.979855	.65	.493854	7.42	.506146	41
20	.474115	6.73	.979816	.67	.494299	7.40	.505701	40
21	9.474519	6.73	9.979776	.65	9.494743	7.38	10.505257	39
22	.474923	6.73	.979737	.67	.495186	7.40	.504814	38
23	.475327	6.72	.979697	.65	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.67	.496073	7.37	.503927	36
25	.476133	6.72	.979618	.65	.496515	7.37	.503485	35
26	.476536	6.70	.979579	.67	.496957	7.37	.503043	34
27	.476938	6.70	.979539	.67	.497399	7.37	.502601	33
28	.477340	6.68	.979499	.67	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.65	.498282	7.33	.501718	31
30	.478142	6.67	.979420	.67	.498722	7.35	.501278	30
31	9.478542	6.67	9.979380	.67	9.499163	7.33	10.500837	29
32	.478942	6.67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.32	.499519	26
35	.480140	6.65	.979220	.67	.500920	7.32	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.67	.502235	7.28	.497765	22
39	.481731	6.62	.979059	.68	.502672	7.28	.497328	21
40	.482128	6.62	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6.60	9.978979	.67	9.503546	7.27	10.496454	19
42	.482921	6.58	.978939	.68	.503982	7.27	.496018	18
43	.483316	6.60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6.58	.978858	.68	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.25	.494276	14
47	.484895	6.57	.978737	.68	.506159	7.23	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.67	.507027	7.23	.492973	11
50	.486075	6.53	.978615	.68	.507460	7.22	.492540	10
51	9.486467	6.55	9.978574	.68	9.507893	7.22	10.492107	9
52	.486860	6.52	.978533	.67	.508326	7.22	.491674	8
53	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	.68	.509191	7.18	.490809	6
55	.488034	6.50	.978411	.68	.509622	7.20	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
58	.489204	6.48	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60	9.489982		9.978206	.68	9.511776		10.488224	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.489982	6.48	9.978206	.68	9.511776	7.17	10.488224	60
1	.490371	6.47	.978165	.68	.512206	7.15	.487794	59
2	.490759	6.47	.978124	.68	.512635	7.15	.487365	58
3	.491147	6.47	.978083	.68	.513064	7.15	.486936	57
4	.491535	6.45	.978042	.68	.513493	7.13	.486507	56
5	.491922	6.43	.978001	.70	.513921	7.13	.486079	55
6	.492308	6.45	.977959	.68	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.68	.514777	7.12	.485223	53
8	.493081	6.42	.977877	.70	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.68	.515631	7.10	.484369	51
10	.493851	6.42	.977794	.70	.516057	7.12	.483943	50
11	9.494236	6.42	9.977752	.68	9.516484	7.10	10.483516	49
12	.494621	6.40	.977711	.70	.516910	7.08	.483090	48
13	.495005	6.38	.977669	.68	.517335	7.10	.482665	47
14	.495388	6.40	.977628	.70	.517761	7.08	.482239	46
15	.495772	6.37	.977586	.70	.518186	7.07	.481814	45
16	.496154	6.38	.977544	.68	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.37	.977461	.70	.519458	7.07	.480542	42
19	.497301	6.35	.977419	.70	.519882	7.05	.480118	41
20	.497682	6.35	.977377	.70	.520305	7.05	.479695	40
21	9.498064	6.33	9.977335	.70	9.520728	7.05	10.479272	39
22	.498444	6.35	.977293	.70	.521151	7.03	.478849	38
23	.498825	6.32	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.32	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.32	.977083	.70	.523259	7.02	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.00	.476320	32
29	.501099	6.28	.976999	.70	.524100	7.00	.475900	31
30	.501476	6.30	.976957	.72	.524520	7.00	.475480	30
31	9.501854	6.28	9.976914	.70	9.524940	6.98	10.475060	29
32	.502231	6.27	.976872	.70	.525359	6.98	.474641	28
33	.502607	6.28	.976830	.72	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.70	.526197	6.97	.473803	26
35	.503360	6.25	.976745	.72	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.70	.527033	6.97	.472967	24
37	.504110	6.25	.976660	.72	.527451	6.95	.472549	23
38	.504485	6.25	.976617	.72	.527868	6.95	.472132	22
39	.504860	6.23	.976574	.70	.528285	6.95	.471715	21
40	.505234	6.23	.976532	.72	.528702	6.95	.471298	20
41	9.505608	6.22	9.976489	.72	9.529119	6.93	10.470881	19
42	.505981	6.22	.976446	.70	.529535	6.93	.470465	18
43	.506354	6.22	.976404	.72	.529951	6.92	.470049	17
44	.506727	6.20	.976361	.72	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.92	.469219	15
46	.507471	6.20	.976275	.72	.531196	6.92	.468804	14
47	.507843	6.18	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.90	.467561	11
50	.508956	6.17	.976103	.72	.532853	6.88	.467147	10
51	9.509326	6.17	9.976060	.72	9.533266	6.88	10.466734	9
52	.509696	6.15	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.73	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.15	.975887	.72	.534916	6.87	.465084	5
56	.511172	6.13	.975844	.73	.535328	6.85	.464672	4
57	.511540	6.12	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.73	.536561	6.85	.463439	1
60	9.512642		9.975670		9.536972		10.463028	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.512642	6.12	9.975670	.72	9.536972	6.83	10.463028	60
1	.513009	6.10	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.72	.538202	6.82	.461798	57
4	.514107	6.08	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6.82	.460980	55
6	.514837	6.08	.975408	.73	.539429	6.80	.460571	54
7	.515202	6.07	.975365	.72	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6.80	.459347	51
10	.516294	6.05	.975233	.73	.541061	6.78	.458939	50
11	9.516657	6.05	9.975189	.73	9.541468	6.78	10.458532	49
12	.517020	6.03	.975145	.73	.541875	6.77	.458125	48
13	.517382	6.05	.975101	.73	.542281	6.78	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.02	.975013	.73	.543094	6.75	.456906	45
16	.518468	6.02	.974969	.73	.543499	6.77	.456501	44
17	.518829	6.02	.974925	.73	.543905	6.75	.456095	43
18	.519190	6.02	.974880	.75	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.73	.544715	6.73	.455285	41
20	.519911	6.00	.974792	.73	.545119	6.75	.454881	40
21	9.520271	6.00	9.974748	.75	9.545524	6.73	10.454476	39
22	.520631	5.98	.974703	.73	.545928	6.72	.454072	38
23	.520990	5.98	.974659	.75	.546331	6.73	.453669	37
24	.521349	5.97	.974614	.75	.546735	6.72	.453265	36
25	.521707	5.98	.974570	.73	.547138	6.70	.452862	35
26	.522066	5.97	.974525	.75	.547540	6.72	.452460	34
27	.522424	5.95	.974481	.73	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.75	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.75	.548747	6.70	.451253	31
30	.523495	5.95	.974347	.75	.549149	6.68	.450851	30
31	9.523852	5.93	9.974302	.75	9.549550	6.68	10.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.68	.449248	26
35	.525275	5.92	.974122	.75	.551153	6.65	.448847	25
36	.525630	5.90	.974077	.75	.551552	6.67	.448448	24
37	.525984	5.92	.974032	.75	.551952	6.65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.88	.973942	.75	.552750	6.65	.447250	21
40	.527046	5.90	.973897	.75	.553149	6.65	.446851	20
41	9.527400	5.88	9.973852	.75	9.553548	6.63	10.446452	19
42	.527753	5.87	.973807	.75	.553946	6.63	.446054	18
43	.528105	5.88	.973761	.75	.554344	6.62	.445656	17
44	.528458	5.87	.973716	.75	.554741	6.63	.445259	16
45	.528810	5.85	.973671	.75	.555139	6.62	.444861	15
46	.529161	5.87	.973625	.75	.555536	6.62	.444464	14
47	.529513	5.85	.973580	.75	.555933	6.60	.444067	13
48	.529864	5.85	.973535	.77	.556329	6.60	.443671	12
49	.530215	5.83	.973489	.75	.556725	6.60	.443275	11
50	.530565	5.83	.973444	.77	.557121	6.60	.442879	10
51	9.530915	5.83	9.973398	.77	9.557517	6.60	10.442483	9
52	.531265	5.82	.973352	.77	.557913	6.58	.442087	8
53	.531614	5.82	.973307	.77	.558308	6.58	.441692	7
54	.531963	5.82	.973261	.77	.558708	6.57	.441297	6
55	.532312	5.82	.973215	.77	.559097	6.57	.440903	5
56	.532661	5.80	.973169	.77	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.77	.559885	6.57	.440115	3
58	.533357	5.78	.973078	.77	.560279	6.57	.439721	2
59	.533704	5.80	.973032	.77	.560673	6.55	.439327	1
60	9.534052		9.972986		9.561066		10.438934	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.534052		9.972986		9.561066		10.438934	60
1	.534399	5.78	.972940	.77	.561459	6.55	.438541	59
2	.534745	5.77	.972894	.77	.561851	6.53	.438149	58
3	.535092	5.78	.972848	.77	.562244	6.55	.437756	57
4	.535438	5.77	.972802	.77	.562636	6.53	.437364	56
5	.535783	5.75	.972755	.78	.563028	6.53	.436972	55
6	.536129	5.77	.972709	.77	.563419	6.52	.436581	54
7	.536474	5.75	.972663	.77	.563811	6.53	.436189	53
8	.536818	5.73	.972617	.77	.564202	6.52	.435798	52
9	.537163	5.75	.972570	.78	.564593	6.52	.435407	51
10	.537507	5.73	.972524	.77	.564983	6.50	.435017	50
		5.73		.77		6.50		
11	9.537851		9.972478		9.565373		10.434627	49
12	.538194	5.72	.972431	.78	.565763	6.50	.434237	48
13	.538538	5.70	.972385	.77	.566153	6.50	.433847	47
14	.538880	5.70	.972338	.78	.566542	6.48	.433458	46
15	.539223	5.72	.972291	.78	.566932	6.50	.433068	45
16	.539565	5.70	.972245	.77	.567320	6.47	.432680	44
17	.539907	5.70	.972198	.78	.567709	6.48	.432291	43
18	.540249	5.68	.972151	.78	.568098	6.48	.431902	42
19	.540590	5.68	.972105	.77	.568486	6.47	.431514	41
20	.540931	5.68	.972058	.78	.568873	6.45	.431127	40
		5.68		.78		6.47		
21	9.541272		9.972011		9.569261		10.430739	39
22	.541613	5.68	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.67	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.67	.971870	.78	.570422	6.45	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.43	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.65	.971729	.78	.571581	6.43	.428419	33
28	.543649	5.65	.971682	.78	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.78	.572352	6.43	.427648	31
30	.544325	5.63	.971588	.80	.572738	6.42	.427262	30
		5.63						
31	9.544663		9.971540	.78	9.573123	6.40	10.426877	29
32	.545000	5.62	.971493	.78	.573507	6.42	.426493	28
33	.545338	5.63	.971446	.80	.573892	6.40	.426108	27
34	.545674	5.60	.971398	.78	.574276	6.40	.425724	26
35	.546011	5.62	.971351	.80	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.78	.575044	6.38	.424956	24
37	.546683	5.60	.971256	.80	.575427	6.38	.424573	23
38	.547019	5.60	.971208	.78	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.80	.576193	6.38	.423807	21
40	.547689	5.58	.971113	.78	.576576	6.38	.423424	20
		5.58						
41	9.548024		9.971066	.80	9.576959	6.37	10.423041	19
42	.548359	5.58	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.57	.970970	.80	.577723	6.35	.422277	17
44	.549027	5.57	.970922	.80	.578104	6.37	.421896	16
45	.549360	5.55	.970874	.78	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.35	.420752	13
48	.550359	5.55	.970731	.80	.579629	6.33	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.33	.419991	11
50	.551024	5.53	.970635	.82	.580389	6.33	.419611	10
51	9.551356		9.970586	.80	9.580769	6.33	10.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.52	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.50	.970394	.82	.582286	6.32	.417714	5
56	.553010	5.52	.970345	.80	.582665	6.32	.417335	4
57	.553341	5.48	.970297	.80	.583044	6.32	.416956	3
58	.553670	5.50	.970249	.82	.583422	6.30	.416578	2
59	.554000	5.48	.970200	.80	.583800	6.30	.416200	1
60	9.554329		9.970152		9.584177	6.28	10.415823	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.554329		9.970152		9.584177		10.415823	60
1	.554658	5.48	.970103	.82	.584555	6.30	.415445	59
2	.554987	5.48	.970055	.80	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.82	.585686	6.28	.414314	56
5	.555971	5.47	.969909	.80	.586062	6.27	.413938	55
6	.556299	5.47	.969860	.82	.586439	6.28	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.27	.413185	53
8	.556953	5.45	.969762	.82	.587190	6.25	.412810	52
9	.557280	5.45	.969714	.80	.587566	6.27	.412434	51
10	.557606	5.43	.969665	.82	.587941	6.25	.412059	50
11	9.557932		9.969616		9.588316		10.411684	49
12	.558258	5.43	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.42	.969518	.82	.589066	6.25	.410934	47
14	.558909	5.43	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.42	.969420	.82	.589814	6.23	.410186	45
16	.559558	5.40	.969370	.83	.590188	6.23	.409812	44
17	.559883	5.42	.969321	.82	.590562	6.23	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.82	.591308	6.22	.408692	41
20	.560855	5.40	.969173	.83	.591681	6.22	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.22	10.407946	39
22	.561501	5.38	.969075	.83	.592426	6.20	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.22	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.20	.406829	36
25	.562468	5.37	.968926	.82	.593542	6.18	.406458	35
26	.562790	5.37	.968877	.83	.593914	6.20	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	.564075	5.35	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.33	9.968628	.83	9.595768	6.17	10.404232	29
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	28
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	27
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	23
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	22
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	21
40	.567269	5.30	.968178	.83	.599091	6.13	.400909	20
41	9.567587	5.28	9.968128	.83	9.599459	6.13	10.400541	19
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	18
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	17
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	16
45	.568856	5.27	.967927	.83	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.85	.601296	6.12	.398704	14
47	.569488	5.27	.967826	.83	.601663	6.10	.398337	13
48	.569804	5.27	.967775	.85	.602029	6.10	.397971	12
49	.570120	5.25	.967725	.83	.602395	6.10	.397605	11
50	.570435	5.27	.967674	.85	.602761	6.10	.397239	10
51	9.570751	5.25	9.967624	.85	9.603127	6.10	10.396873	9
52	.571066	5.23	.967573	.85	.603493	6.08	.396507	8
53	.571380	5.25	.967522	.85	.603858	6.08	.396142	7
54	.571695	5.23	.967471	.83	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.22	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.23	.967319	.85	.605317	6.08	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.20	.967217	.85	.606046	6.07	.393954	1
60	9.573575		9.967166		9.606410		10.393590	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.573575	5.22	9.967166	.85	9.606410	6.05	10.393590	60
1	.573888	5.20	.967115	.85	.606773	6.07	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.87	.607500	6.05	.392500	57
4	.574824	5.20	.966961	.85	.607863	6.03	.392137	56
5	.575136	5.18	.966910	.85	.608225	6.05	.391775	55
6	.575447	5.18	.966859	.85	.608588	6.03	.391412	54
7	.575758	5.18	.966808	.87	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.85	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.87	.609674	6.03	.390326	51
10	.576689	5.17	.966653	.85	.610036	6.02	.389964	50
11	9.576999	5.17	9.966602	.87	9.610397	6.03	10.389603	49
12	.577309	5.15	.966550	.85	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.87	.611120	6.00	.388880	47
14	.577927	5.15	.966447	.87	.611480	6.02	.388520	46
15	.578236	5.15	.966395	.85	.611841	6.00	.388159	45
16	.578545	5.13	.966344	.87	.612201	6.00	.387799	44
17	.578853	5.15	.966292	.87	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.87	.612921	6.00	.387079	42
19	.579470	5.12	.966188	.87	.613281	6.00	.386719	41
20	.579777	5.13	.966136	.85	.613641	5.98	.386359	40
21	9.580085	5.12	9.966085	.87	9.614000	5.98	10.386000	39
22	.580392	5.12	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.10	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.12	.965929	.88	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.08	.965772	.87	.616151	5.97	.383849	33
28	.582229	5.10	.965720	.87	.616509	5.97	.383491	32
29	.582535	5.08	.965668	.88	.616867	5.95	.383133	31
30	.582840	5.08	.965615	.87	.617224	5.97	.382776	30
31	9.583145	5.07	9.965563	.87	9.617582	5.95	10.382418	29
32	.583449	5.08	.965511	.88	.617939	5.93	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.95	.381705	27
34	.584058	5.05	.965406	.88	.918652	5.93	.381348	26
35	.584361	5.07	.965353	.87	.619008	5.93	.380992	25
36	.584665	5.05	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.07	.965248	.88	.619720	5.93	.380280	23
38	.585272	5.03	.965195	.87	.620076	5.93	.379924	22
39	.585574	5.05	.965143	.88	.620432	5.92	.379568	21
40	.585877	5.03	.965090	.88	.620787	5.92	.379213	20
41	9.586179	5.05	9.965037	.88	9.621142	5.92	10.378858	19
42	.586482	5.02	.964984	.88	.621497	5.92	.378503	18
43	.586783	5.03	.964931	.87	.621852	5.92	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.90	.377793	16
45	.587386	5.03	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.02	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.00	.964720	.90	.623269	5.90	.376731	13
48	.588289	5.02	.964666	.88	.623623	5.88	.376377	12
49	.588590	5.00	.964613	.88	.623976	5.90	.376024	11
50	.588890	5.00	.964560	.88	.624330	5.88	.375670	10
51	9.589190	4.98	9.964507	.88	9.624683	5.88	10.375317	9
52	.589489	5.00	.964454	.90	.625036	5.87	.374964	8
53	.589789	4.98	.964400	.88	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.88	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.90	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.88	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.90	.626797	5.87	.373203	3
58	.591282	4.97	.964133	.88	.627149	5.87	.372851	2
59	.591580	4.97	.964080	.90	.627501	5.85	.372499	1
60	9.591878		9.964026		9.627852		10.372148	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.591878	4.97	9.964026	.90	9.627852	5.85	10.372148	60
1	.592176	4.95	.963972	.88	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.83	.371095	57
4	.593067	4.93	.963811	.90	.629255	5.85	.370745	56
5	.593363	4.93	.963757	.88	.629606	5.83	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.82	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.83	.368995	51
10	.594842	4.92	.963488	.90	.631355	5.82	.368645	50
11	9.595137	4.92	9.963434	.92	9.631704	5.82	10.368296	49
12	.595432	4.92	.963379	.90	.632053	5.82	.367947	48
13	.595727	4.90	.963325	.90	.632402	5.80	.367598	47
14	.596021	4.90	.963271	.90	.632750	5.82	.367250	46
15	.596315	4.90	.963217	.90	.633099	5.80	.366901	45
16	.596609	4.90	.963163	.92	.633447	5.80	.366553	44
17	.596903	4.88	.963108	.90	.633795	5.80	.366205	43
18	.597196	4.90	.963054	.92	.634143	5.78	.365857	42
19	.597490	4.88	.962999	.90	.634490	5.80	.365510	41
20	.597783	4.87	.962945	.92	.634838	5.78	.365162	40
21	9.598075	4.88	9.962890	.90	9.635185	5.78	10.364815	39
22	.598368	4.87	.962836	.92	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.90	.635879	5.78	.364121	37
24	.598952	4.87	.962727	.92	.636226	5.77	.363774	36
25	.599244	4.87	.962672	.92	.636572	5.78	.363428	35
26	.599536	4.85	.962617	.92	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.90	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.92	.637611	5.75	.362389	32
29	.600409	4.85	.962453	.92	.637956	5.77	.362044	31
30	.600700	4.83	.962398	.92	.638302	5.75	.361698	30
31	9.600990	4.83	9.962343	.92	9.638647	5.75	10.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.75	.360318	26
35	.602150	4.82	.962123	.93	.640027	5.73	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.75	.359629	24
37	.602728	4.82	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.80	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.82	.961902	.93	.641404	5.72	.358596	21
40	.603594	4.80	.961846	.92	.641747	5.73	.358253	20
41	9.603882	4.80	9.961791	.93	9.642091	5.72	10.357909	19
42	.604170	4.78	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.80	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.78	.961624	.92	.643120	5.72	.356880	16
45	.605032	4.78	.961569	.93	.643462	5.72	.356537	15
46	.605319	4.78	.961513	.92	.643806	5.70	.356194	14
47	.605606	4.77	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.78	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	.606465	4.77	.961290	.92	.645174	5.70	.354826	10
51	9.606751	4.75	9.961235	.93	9.645516	5.68	10.354484	9
52	.607036	4.77	.961179	.93	.645857	5.70	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.68	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.75	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.73	.960955	.93	.647222	5.67	.352778	4
57	.608461	4.73	.960899	.93	.647562	5.68	.352438	3
58	.608745	4.73	.960843	.95	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.93	.648243	5.67	.351757	1
60	9.609313		9.960730		9.648583		10.351417	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.609313	4.73	9.960730	.93	9.648583	5.67	10.351417	60
1	.609597	4.72	.960674	.93	.648923	5.67	.351077	59
2	.609880	4.73	.960618	.93	.649263	5.65	.350737	58
3	.610164	4.72	.960561	.95	.649602	5.67	.350398	57
4	.610447	4.70	.960505	.93	.649942	5.65	.350058	56
5	.610729	4.72	.960448	.95	.650281	5.65	.349719	55
6	.611012	4.70	.960392	.93	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.95	.650959	5.63	.349041	53
8	.611576	4.70	.960279	.93	.651297	5.65	.348703	52
9	.611858	4.70	.960222	.95	.651636	5.63	.348364	51
10	.612140	4.68	.960165	.93	.651974	5.63	.348026	50
11	9.612421	4.68	9.960109	.95	9.652312	5.63	10.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.93	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.67	.959711	.95	.654674	5.62	.345326	42
19	.614665	4.65	.959654	.97	.655011	5.62	.344989	41
20	.614944	4.65	.959596	.95	.655348	5.60	.344652	40
21	9.615223	4.65	9.959539	.95	9.655684	5.60	10.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.65	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.63	.959368	.97	.656692	5.60	.343308	36
25	.616338	4.63	.959310	.95	.657028	5.60	.342972	35
26	.616616	4.63	.959253	.97	.657364	5.58	.342636	34
27	.616894	4.63	.959195	.95	.657699	5.58	.342301	33
28	.617172	4.63	.959138	.97	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.95	.658369	5.58	.341631	31
30	.617727	4.62	.959023	.97	.658704	5.58	.341296	30
31	9.618004	4.62	9.958965	.95	9.659039	5.57	10.340961	29
32	.618281	4.62	.958908	.97	.659373	5.58	.340627	28
33	.618558	4.60	.958850	.97	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.97	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.95	.660376	5.57	.339624	25
36	.619386	4.60	.958677	.97	.660710	5.55	.339290	24
37	.619662	4.60	.958619	.97	.661043	5.57	.338957	23
38	.619938	4.58	.958561	.97	.661377	5.55	.338623	22
39	.620213	4.58	.958503	.97	.661710	5.55	.338290	21
40	.620488	4.58	.958445	.97	.662043	5.55	.337957	20
41	9.620763	4.58	9.958387	.97	9.662376	5.55	10.337624	19
42	.621038	4.58	.958329	.97	.662709	5.55	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.55	.336958	17
44	.621587	4.57	.958213	.98	.663375	5.53	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.53	.336293	15
46	.622135	4.57	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.55	.958038	.98	.664371	5.53	.335629	13
48	.622682	4.57	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.52	.334965	11
50	.623229	4.55	.957863	.98	.665366	5.53	.334634	10
51	9.623502	4.53	9.957804	.97	9.665698	5.52	10.334302	9
52	.623774	4.55	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.53	.957687	.98	.666360	5.52	.333640	7
54	.624319	4.53	.957628	.97	.666691	5.50	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.52	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.50	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.52	.332318	3
58	.625406	4.52	.957393	.97	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	9.625948		9.957276		9.668673		10.331327	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.625948	4.52	9.957276	.98	9.668673	5.48	10.331327	60
1	.626219	4.52	.957177	.98	.669002	5.50	.330998	59
2	.626490	4.50	.957158	.98	.669332	5.48	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.50	.330339	57
4	.627030	4.50	.957040	.98	.669991	5.48	.330009	56
5	.627300	4.50	.956981	1.00	.670320	5.48	.329680	55
6	.627570	4.50	.956921	.98	.670649	5.47	.329351	54
7	.627840	4.48	.956862	.98	.670977	5.48	.329023	53
8	.628109	4.48	.956803	.98	.671306	5.48	.328694	52
9	.628378	4.48	.956744	1.00	.671635	5.47	.328365	51
10	.628647	4.48	.956684	.98	.671963	5.47	.328037	50
11	9.628916	4.48	9.956625	.98	9.672291	5.47	10.327709	49
12	.629185	4.47	.956566	1.00	.672619	5.47	.327381	48
13	.629453	4.47	.956506	.98	.672947	5.45	.327053	47
14	.629721	4.47	.956447	1.00	.673274	5.47	.326726	46
15	.629989	4.47	.956387	1.00	.673602	5.45	.326398	45
16	.630257	4.45	.956327	.98	.673929	5.47	.326071	44
17	.630524	4.47	.956268	1.00	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	.98	.674911	5.43	.325089	41
20	.631326	4.45	.956089	1.00	.675237	5.45	.324763	40
21	9.631593	4.43	9.956029	1.00	9.675564	5.43	10.324436	39
22	.631859	4.43	.955969	1.00	.675890	5.45	.324110	38
23	.632125	4.45	.955909	1.00	.676217	5.43	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.42	.955789	1.00	.676869	5.42	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.43	.322480	33
28	.633454	4.42	.955609	.98	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	.633984	4.42	.955488	1.00	.678496	5.42	.321504	30
31	9.634249	4.42	9.955428	1.00	9.678821	5.42	10.321179	29
32	.634514	4.40	.955368	1.02	.679146	5.42	.320854	28
33	.634778	4.40	.955307	1.00	.679471	5.40	.320529	27
34	.635042	4.40	.955247	1.02	.679795	5.42	.320205	26
35	.635306	4.40	.955186	1.00	.680120	5.40	.319880	25
36	.635570	4.40	.955126	1.02	.680444	5.40	.319556	24
37	.635834	4.38	.955065	1.00	.680768	5.40	.319232	23
38	.636097	4.38	.955005	1.02	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.02	.681416	5.40	.318584	21
40	.636623	4.38	.954883	1.00	.681740	5.38	.318260	20
41	9.636886	4.37	9.954823	1.02	9.682063	5.40	10.317937	19
42	.637148	4.38	.954762	1.02	.682387	5.38	.317613	18
43	.637411	4.37	.954701	1.02	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.37	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.35	.954518	1.02	.683679	5.37	.316321	14
47	.638458	4.37	.954457	1.02	.684001	5.38	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	.639242	4.35	.954274	1.02	.684968	5.37	.315032	10
51	9.639503	4.35	9.954213	1.02	9.685290	5.37	10.314710	9
52	.639764	4.33	.954152	1.03	.685612	5.37	.314388	8
53	.640024	4.33	.954090	1.02	.685934	5.35	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.37	.313745	6
55	.640544	4.33	.953968	1.03	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.33	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.02	.687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
60	9.641842		9.953660		9.688182		10.311818	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.825511	2.33	9.871073	1.88	9.954437	4.23	10.045563	60
1	.825651	2.33	.870960	1.90	.954691	4.25	.045309	59
2	.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955708	4.22	.044292	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.92	.956215	4.23	.043785	53
8	.826631	2.32	.870161	1.90	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.90	.956723	4.23	.043277	51
10	.826910	2.32	.869933	1.92	.956977	4.23	.043023	50
11	9.827049	2.32	9.869818	1.90	9.957231	4.23	10.042769	49
12	.827189	2.32	.869704	1.92	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.92	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.90	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.92	.958247	4.22	.041753	45
16	.827745	2.32	.869245	1.92	.958500	4.23	.041500	44
17	.827884	2.32	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.32	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.32	.868900	1.92	.959262	4.23	.040738	41
20	.828301	2.30	.868785	1.92	.959516	4.22	.040484	40
21	9.828439	2.32	9.868670	1.92	9.959769	4.23	10.040231	39
22	.828578	2.30	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.32	.868440	1.93	.960277	4.22	.039723	37
24	.828855	2.30	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.93	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.22	.038708	33
28	.829407	2.30	.867862	1.92	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.22	.038201	31
30	.829683	2.30	.867631	1.93	.962052	4.23	.037948	30
31	9.829821	2.30	9.867515	1.93	9.962306	4.23	10.037694	29
32	.829959	2.30	.867399	1.93	.962560	4.22	.037440	28
33	.830097	2.28	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.30	.867167	1.93	.963067	4.22	.036933	26
35	.830372	2.28	.867051	1.93	.963320	4.23	.036680	25
36	.830509	2.28	.866935	1.93	.963574	4.23	.036426	24
37	.830646	2.28	.866819	1.93	.963828	4.22	.036172	23
38	.830784	2.30	.866703	1.95	.964081	4.23	.035919	22
39	.830921	2.28	.866586	1.93	.964335	4.22	.035665	21
40	.831058	2.28	.866470	1.95	.964588	4.23	.035412	20
41	9.831195	2.28	9.866353	1.93	9.964842	4.22	10.035158	19
42	.831332	2.28	.866237	1.95	.965095	4.23	.034905	18
43	.831469	2.28	.866120	1.93	.965349	4.22	.034651	17
44	.831606	2.27	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.23	.034145	15
46	.831879	2.27	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.28	.865653	1.95	.966362	4.23	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.28	.865419	1.95	.966869	4.23	.033131	11
50	.832425	2.27	.865302	1.95	.967123	4.22	.032877	10
51	9.832561	2.27	9.865185	1.95	9.967376	4.22	10.032624	9
52	.832697	2.27	.865068	1.97	.967629	4.23	.032371	8
53	.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.95	.968136	4.22	.031864	6
55	.833105	2.27	.864716	1.97	.968389	4.23	.031611	5
56	.833241	2.27	.864598	1.95	.968643	4.22	.031357	4
57	.833377	2.25	.864481	1.97	.968896	4.22	.031104	3
58	.833512	2.27	.864363	1.97	.969149	4.23	.030851	2
59	.833648	2.25	.864245	1.97	.969403	4.22	.030597	1
60	9.833783		9.864127		9.969656		10.030344	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.833783	2.27	9.864127	1.95	9.969656	4.22	10.030344	60
1	.833919	2.25	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.23	.029838	58
3	.834189	2.27	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.98	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.23	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.23	.863183	1.98	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	.835134	2.25	.862946	1.98	.972188	4.22	.027812	50
11	9.835269	2.23	9.862827	1.97	9.972441	4.23	10.027559	49
12	.835403	2.25	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.23	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.25	.862471	1.97	.973201	4.22	.026799	46
15	.835807	2.23	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.23	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.23	.025534	41
20	.836477	2.23	.861758	2.00	.974720	4.22	.025280	40
21	9.836611	2.23	9.861638	1.98	9.974973	4.22	10.025027	39
22	.836745	2.22	.861519	1.98	.975226	4.22	.024774	38
23	.836878	2.23	.861400	2.00	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.98	.975732	4.22	.024268	36
25	.837146	2.22	.861161	2.00	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.98	.976238	4.22	.023762	34
27	.837412	2.23	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	.837812	2.22	.860562	2.00	.977250	4.22	.022750	30
31	9.837945	2.22	9.860442	2.00	9.977503	4.22	10.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.22	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.22	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.22	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.20	.859842	2.02	.978768	4.22	.021232	24
37	.838742	2.22	.859721	2.00	.979021	4.22	.020979	23
38	.838875	2.20	.859601	2.02	.979274	4.22	.020726	22
39	.839007	2.22	.859480	2.00	.979527	4.22	.020473	21
40	.839140	2.20	.859360	2.02	.979780	4.22	.020220	20
41	9.839272	2.20	9.859239	2.00	9.980033	4.22	10.019967	19
42	.839404	2.20	.859119	2.02	.980286	4.20	.019714	18
43	.839536	2.20	.858998	2.02	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.22	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.22	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.22	.018450	13
48	.840196	2.20	.858393	2.02	.981803	4.22	.018197	12
49	.840328	2.18	.858272	2.02	.982056	4.22	.017944	11
50	.840459	2.20	.858151	2.03	.982309	4.22	.017691	10
51	9.840591	2.18	9.858029	2.02	9.982562	4.20	10.017438	9
52	.840722	2.20	.857908	2.03	.982814	4.22	.017186	8
53	.840854	2.18	.857786	2.02	.983067	4.22	.016933	7
54	.840985	2.18	.857665	2.03	.983320	4.22	.016680	6
55	.841116	2.18	.857543	2.02	.983573	4.22	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.22	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.22	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.20	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.22	.015416	1
60	9.841771		9.856934		9.984837		10.015163	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.841771	2.18	9.856934	2.03	9.984837	4.22	10.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.22	.014910	59
2	.842033	2.17	.856690	2.03	.985343	4.22	.014657	58
3	.842163	2.18	.856568	2.03	.985596	4.20	.014404	57
4	.842294	2.17	.856446	2.05	.985848	4.22	.014152	56
5	.842424	2.18	.856323	2.03	.986101	4.22	.013899	55
6	.842555	2.17	.856201	2.05	.986354	4.22	.013646	54
7	.842685	2.17	.856078	2.03	.986607	4.22	.013393	53
8	.842815	2.18	.855956	2.05	.986860	4.20	.013140	52
9	.842946	2.17	.855833	2.03	.987112	4.22	.012888	51
10	.843076	2.17	.855711	2.05	.987365	4.22	.012635	50
11	9.843206	2.17	9.855588	2.05	9.987618	4.22	10.012382	49
12	.843336	2.17	.855465	2.05	.987871	4.20	.012129	48
13	.843466	2.15	.855342	2.05	.988123	4.22	.011877	47
14	.843595	2.17	.855219	2.05	.988376	4.22	.011624	46
15	.843725	2.17	.855096	2.05	.988629	4.22	.011371	45
16	.843855	2.15	.854973	2.05	.988882	4.20	.011118	44
17	.843984	2.17	.854850	2.05	.989134	4.22	.010866	43
18	.844114	2.15	.854727	2.07	.989387	4.22	.010613	42
19	.844243	2.15	.854603	2.05	.989640	4.22	.010360	41
20	.844372	2.17	.854480	2.07	.989893	4.20	.010107	40
21	9.844502	2.15	9.854356	2.05	9.990145	4.22	10.009855	39
22	.844631	2.15	.854233	2.07	.990398	4.22	.009602	38
23	.844760	2.15	.854109	2.05	.990651	4.20	.009349	37
24	.844889	2.15	.853986	2.07	.990903	4.22	.009097	36
25	.845018	2.15	.853862	2.07	.991156	4.22	.008844	35
26	.845147	2.15	.853738	2.07	.991409	4.22	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.20	.008338	33
28	.845405	2.13	.853490	2.07	.991914	4.22	.008086	32
29	.845533	2.15	.853366	2.07	.992167	4.22	.007833	31
30	.845662	2.13	.853242	2.07	.992420	4.20	.007580	30
31	9.845790	2.15	9.853118	2.07	9.992672	4.22	10.007328	29
32	.845919	2.13	.852994	2.08	.992925	4.22	.007075	28
33	.846047	2.13	.852869	2.07	.993178	4.22	.006822	27
34	.846175	2.15	.852745	2.08	.993431	4.20	.006569	26
35	.846304	2.13	.852620	2.07	.993683	4.22	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.22	.006064	24
37	.846560	2.13	.852371	2.07	.994189	4.20	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.22	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.22	.005306	21
40	.846944	2.12	.851997	2.08	.994947	4.20	.005053	20
41	9.847071	2.13	9.851872	2.08	9.995199	4.22	10.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.22	.004548	18
43	.847327	2.12	.851622	2.08	.995705	4.20	.004295	17
44	.847454	2.13	.851497	2.08	.995957	4.22	.004043	16
45	.847582	2.12	.851372	2.10	.996210	4.22	.003790	15
46	.847709	2.12	.851246	2.08	.996463	4.20	.003537	14
47	.847836	2.13	.851121	2.08	.996715	4.22	.003285	13
48	.847964	2.12	.850996	2.10	.996968	4.22	.003032	12
49	.848091	2.12	.850870	2.08	.997221	4.20	.002779	11
50	.848218	2.12	.850745	2.10	.997473	4.22	.002527	10
51	9.848345	2.12	9.850619	2.10	9.997726	4.22	10.002274	9
52	.848472	2.12	.850493	2.08	.997979	4.20	.002021	8
53	.848599	2.12	.850368	2.10	.998231	4.22	.001769	7
54	.848726	2.10	.850242	2.10	.998484	4.22	.001516	6
55	.848852	2.12	.850116	2.10	.998737	4.20	.001263	5
56	.848979	2.12	.849990	2.10	.998989	4.22	.001011	4
57	.849106	2.10	.849864	2.10	.999242	4.22	.000758	3
58	.849232	2.12	.849738	2.12	.999495	4.20	.000505	2
59	.849359	2.10	.849611	2.10	.999747	4.22	.000253	1
60	9.849485		9.849485		10.000000		10.000000	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

NATURAL FUNCTIONS.

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17703	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94205	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45963	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	40
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75903	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74605	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75630	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74352	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20 9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

′	4°		5°		6°		7°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1281	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11[illegible]	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	8.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

′	8°		9°		10°		11°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

′	12°		13°		14°		15°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3 60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

′	16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2 87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

LIBRARY OF CONGRESS
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